NRL REPORT R-3486

ANALYSIS AND CALIBRATION OF LOOP PROBES FOR USE IN MEASURING INTERFERENCE FIELDS

G. A. Morgan, Jr.

June 17, 1949

Approved by:

Mr. F. M. Gager, Head, Special Research Branch
Dr. R. M. Page, Superintendent, Radio Division III

NAVAL RESEARCH LABORATORY
CAPTAIN F. R. FURTH, USN, DIRECTOR
WASHINGTON, D.C.

UNCLASSIFIED

CONTENTS

Abstract iv

Problem Status iv

Authorization iv

INTRODUCTION 1

ANALYSIS OF THE PROBLEM 1

LOOP-ANTENNA THEORY 3

DEVELOPMENT OF A STANDARD RADIO-FREQUENCY FIELD 5

PROOF OF FIELD STRENGTH 8

DETAILS OF CALIBRATING SETUP 11

METHOD OF CALIBRATION 14

DESCRIPTION OF LOOP PROBES 15

RESULTS OF CALIBRATIONS 16

COMMENTS ON LOOP-CALIBRATION RESULTS 21

EFFECT OF TERMINATING RESISTANCE 21

CONCLUSIONS 22

RECOMMENDATIONS 24

APPENDIX A – EQUATIONS FOR IMPEDANCE AND VOLTAGE OF A LOOP PROBE 25

APPENDIX B – VOLTAGE INDUCED IN A LOOP PROBE BY A SMALL RADIATING LOOP ANTENNA 41

APPENDIX C – COMPARISON OF VARIOUS LOOP-ANTENNA EQUATIONS 47

REFERENCES 53

ABSTRACT

A small shielded-loop antenna, to be used as a probe for indicating and measuring radio-frequency interference fields from electronic equipment, has been analyzed. The input impedance is similar to an equivalent shorted two-wire, balanced, transmission line. Voltage induced by a field conforms, over the usable frequency range, to that calculated by equations applicable at low frequencies, but the output voltage varies because of the transmission-line effect.

A number of loop-probe designs have been analyzed and found to have undesirable characteristic impedance discontinuities which make calculation of their characteristics almost impossible. A method of calibration, which uses another small shielded loop to establish a radio-frequency field of known characteristics, has been developed, and its accuracy and reliability are proved experimentally. A simplified loop-probe design has been found to be the most satisfactory. Loop probes of the approximate dimensions of those analyzed have been shown to be usable for the desired application at frequencies below approximately 400 Mc only, both because of unsatisfactory response characteristics and because of difficulties of calibration at higher frequencies.

PROBLEM STATUS

This is a final report on this problem. Unless otherwise advised by the Bureau, the Laboratory will consider the problem closed one month from the mailing date of this report.

AUTHORIZATION

NRL Problem No. R07-04D (BuAer Request No. A282ED)

UNCLASSIFIED

ANALYSIS AND CALIBRATION OF LOOP PROBES FOR USE IN MEASURING INTERFERENCE FIELDS

INTRODUCTION

By Reference 1, the Naval Research Laboratory was requested to "continue the investigations begun by the radio interference group at Radiation Laboratory toward obtaining an analysis of the characteristics of electrostatically shielded loop probe and electrostatic probe pickup devices for use as radio interference field indicating or measuring instruments and developing means of calibrating the devices over the frequency range of 0.1 to 1000 megacycles per second." It was further requested, if investigation proved it feasible, to "develop probes having over-all response characteristics such that they may be used with suitable receivers as radio interference test instruments for simplified field 'go-no go' tests." It was also requested "that evaluation and calibration of the present loop probe AN-3065 be investigated first."

This problem was accepted by the Naval Research Laboratory on June 27, 1946. However, active investigation had not begun when, in December 1946, the Interference Reduction Sub-committee of the Aircraft Radio and Electronics Committee of the Aeronautical Board began preparation of the "Interference Requirements" for a "General Specification for Airborne Electronic Equipment." After several meetings of this sub-committee, general agreement had been reached on all requirements of this portion of the specification with the exception of the section regarding "radiated" interference fields and the pickup devices to be used to measure such fields. In an attempt to provide an answer to this problem, emphasis at the Naval Research Laboratory was placed on investigation of small loop pickup devices and means for calibrating such loop probes. The general problem as stated in Reference 1 was thus restricted in scope. This report covers principally this limited phase of the problem, although some preliminary investigation of the more general problem was begun and is also reported.

In addition, the above-mentioned committee later decided that measurements of interference fields at frequencies lower than 38 megacycles would be made with a rod-type pickup and that the loop probe would be used only for frequencies between 38 and 1000 megacycles. The work at the Naval Research Laboratory was thus further restricted to this frequency range.

ANALYSIS OF THE PROBLEM

In order to determine the preferable type of pickup device, it is necessary to analyze the means by which the fields to be measured may be developed. Assume an electronic device which generates radio-frequency energy of any wave-form, such as continuous wave, pulsed, random noise, or any combination of these types. This electronic device is completely enclosed in a box of copper or aluminum and is connected to an external power supply by unshielded leads. Low-pass radio-frequency filters are connected in the leads inside the box. Small openings are provided through the metallic sides of the box for ventilation. In addition, the box has a poorly fitted lid. Such an elementary electronic device has all the basic deficiencies of the more complex equipment which is to be investigated with a probe.

The probe is to be used to indicate the presence of fields around the equipment; to measure the intensity of any such fields to allow evaluation of the interference which may be produced in other electronic devices located in its immediate vicinity; and, if possible, to determine the point of origin of the fields in order to locate defects in the shielding or filtering.

The elementary electronic device described is to be studied to ascertain how it may develop fields external to the shielded box. This amounts to a review of the principles of shielding, which are discussed in some detail in References 2 and 3. First, it is a basic principle that a radio-frequency field cannot exist inside a perfect conductor. Copper or aluminum are, of course, not perfect conductors but can be considered so for this analysis. Any radio-frequency field which cuts a conductor causes a current to flow in the conductor. This current flow is concentrated on the surface of the conductor and decreases exponentially inside the conductor. Depth of penetration, expressed in inches, is $(2.61 \times 10^{-3})/\sqrt{f}$ in copper or $(3.34 \times 10^{-3})/\sqrt{f}$ in aluminum, where f is the frequency of the field in megacycles. At 38 megacycles, the depth of penetration in copper is 4.24×10^{-4} inch. Copper of 0.0625-inch thickness will produce an attenuation of 1275 db at 38 megacycles. It is obvious that any field of the frequencies of interest in this problem can be considered to be perfectly attenuated by the shield. If the box were a complete enclosure, with no openings, and if there were no conductors passing through the walls, all internal fields would be completely confined.

However, conforming to what is always the case practically, the box is assumed to have some defective joints and openings for ventilation so that the shielding is not perfect. If the openings are small in size compared to the wavelength of the field, the internal field cannot extend directly through the openings to produce external fields. The current produced by the internal fields on the inside wall surfaces will, however, reach the outside surface of the box by flowing along the surface of any break in the shielding. Such current flow of course sets up external fields. The intensity of the current, and consequently the external field, will depend on the shape of the openings and the direction of the internal field so that accurate prediction of the external field strength is virtually impossible. For shield dimensions small compared to wavelength, the field close to the shield surface ($d < \lambda/2\pi$) will be predominantly magnetic since it is caused by current flow. A loop probe, which is considered sensitive to a magnetic field, appears preferable for detecting such fields.

At high frequencies where the shield dimensions exceed a quarter-wavelength, the field may be predominantly electric at some points but will always have a maximum current point at a distance of a quarter-wavelength from the maximum-voltage point, so that a loop probe appears still to be usable. However, at frequencies where the diameter of the loop is approaching a quarter-wavelength, a loop probe may not be satisfactory, since it will extend over a distance which includes both a high current point and a high voltage point, and the maximum field strength will probably not be indicated. A three-inch-diameter loop, as used for AN-3065, has a diameter equal to a quarter-wavelength at approximately 985 megacycles. Consequently, such a loop should be usable at frequencies somewhat lower than 985 megacycles, probably to at least 492 megacycles.

The fields produced by the power leads are difficult to predict, since the leads are equivalent to some form of transmission line and thus the input impedance, at the point where the power leads leave the shield, will vary greatly with frequency. A loop probe will detect any points of high radio-frequency current on the power leads until the loop dimensions become comparable to a quarter-wavelength, and thus such a probe appears satisfactory for the same frequency range as discussed above. In addition, other means are available for measuring radio-frequency current (or voltage) on the power leads, and use of a probe for this purpose is of secondary importance.

LOOP-ANTENNA THEORY

Since loop antennas have been used for direction-finding applications on low frequencies for many years, many detailed studies have been made to determine their characteristics in such uses. Theoretical analyses of such antennas are to be found in numerous places in the literature of radio, for example, in References 4, 5, 6, 7, and 8. Such studies are specifically restricted to low frequencies where the maximum dimensions of the loops are very small compared to the received wavelength. Under such conditions the voltages developed at the loop terminals by radio-frequency fields may be accurately computed.

Detailed development of this theory is given in Appendix A. Briefly, voltage induced in the loop by an r-f field is considered as a lumped voltage applied in series with the inductance, capacity, and resistance of the loop. From a knowledge of these circuit constants, also considered as lumped values, the open-circuit voltage and the impedance at the loop terminals may readily be calculated; and by applying Thevenin's theorem, the voltage developed at the loop terminals may be determined. This method is of such proved accuracy that it is the basis of nearly all procedures for measuring field strength in the low- and medium-frequency ranges. In recent years attempts have been made to broaden the theory to include higher frequencies (for example, References 3 and 9, which are solely in regard to the loop impedance and have explicitly excluded calculation of the induced voltage at higher frequencies, and References 10 and 11, which have attempted to include both impedance and induced voltage).

These developments have been concerned primarily with the use of a loop as an antenna for direction-finding, and further study is required to apply the theory to a loop used as a probe. The loop antenna used for direction-finding is usually tuned to resonance by some tuning element, ordinarily a variable condenser. The loop voltage is commonly applied to a high-impedance circuit, such as the grid of a vacuum tube. The frequency range is invariably restricted, in the order of 3 to 1. A loop probe differs in that it is untuned, is connected to a low-impedance load (a transmission line), and is required to function over a very wide range.* The loop probe may be analyzed on the same basis as the loop antenna, provided the differing conditions are taken into account. At higher frequencies, the circuit constants must be considered as distributed rather than lumped values, and further analysis is necessary.

In all referenced discussions of loop antennas, with the exception of References 10 and 11, the loop and its load have been implicitly assumed to be balanced, that is, both the loop and its load have been assumed to be physically and electrically symmetrical with respect to a plane perpendicular to the plane of the loop and passing midway between its symmetrically located terminals. Since practical requirements for the loop probe at present necessitate use of a coaxial transmission line, the loop probe must be designed so that balanced operation is obtained. The circuits required to connect a balanced loop to an unbalanced transmission line introduce changes in both the open-circuit voltage and loop-terminal impedance. These changes require study.

---

*The range specified for this problem as assigned was 0.1 to 1000 Mc.

Although a number of circuit arrangements for obtaining balanced operation of a loop antenna have been developed for low-frequency applications, only one method, shielding, appears to be practicable for the frequencies at which the loop probe will be used. Such a shielded loop used as an antenna has often been considered the same as an unshielded loop in respect to the voltage induced by a field. Popular explanation of its operation has involved the assumption that an electric field could not but that a magnetic field could penetrate the shield and thereby induce voltage on the inner wire of the loop just as in an unshielded loop. Reference 12 presents one such analysis. But such an explanation is not correct according to basic electromagnetic theory.

An analysis of shielded-loop operation appears in Reference 3, and an extension of this analysis appears in Reference 9. Both analyses point out that a shielded loop is actually a balanced loop consisting of the outside shield surface connected to the output terminals by coaxial line sections, the latter in turn consisting of the leads inside the shield and the inside surfaces of the shield. Reference 9 further converts the balanced loop to an equivalent balanced transmission line section. Reference 11 is another analysis of a shielded loop with a different approach but reaching the same general conclusions.

All these theoretical treatments are for a shielded loop operating into a balanced transmission line. Since the loop probe is connected to a coaxial transmission line, this theory must be further modified. Appendix A contains a development of the theory of the impedance of a shielded loop based on Reference 9 and further extended to apply to a coaxial transmission line load.

As developed in equation 27 of Appendix A, the input impedance of such a loop* is

$$Z_{in} = jZ_{01} \tan\left(\theta_1 + \arctan\left[\frac{Z_{02} \tan\left(\theta_2 + \arctan \dfrac{Z_I + Z_{02} \tan \theta_3}{Z_{02}}\right)}{Z_{01}}\right]\right)$$

where Z_{01} is the characteristic impedance of the "handle,"
Z_{02} is the characteristic impedance of the internal coaxial transmission line sections,
Z_I is the impedance of the loop (shield), and
θ_1, θ_2, and θ_3 are electrical lengths of coaxial line sections.

The characteristic impedances and the electrical lengths are affected by the dielectric constants of the insulating material used to support the center conductor. Calculation or measurement of these various constants is almost impossible, and calculation of the voltage or impedance of the loop based on these constants is, at best, very inaccurate.

Another form of balanced loop antenna operating into a coaxial transmission line is described in Reference 13. The impedance of this type of loop can readily be measured by experimental methods. The equivalent circuit is developed in Appendix A (Figure 28), and an explanation of the method of measuring and calculating its impedance is also given. Because of the circuit complexity of most loop probes, which makes calculation or measurement of circuit constants almost impossible, calibration appeared to require a radio-frequency field having known characteristics. The voltage developed by a loop when located in such a known field can be measured and the calibration determined entirely by experimental means. It is thus established that a radio-frequency field of known intensity is required in order to calibrate a loop probe.

*The resistance of the loop is considered negligible.

DEVELOPMENT OF A STANDARD RADIO-FREQUENCY FIELD

At least two methods have proved satisfactory for developing in the laboratory a radio-frequency field with which to calibrate loop antennas at low frequencies. It was necessary to determine whether either process was satisfactory for use at the frequencies required for calibration of the loop probe. A variation of the method described in References 4 and 5 had been used in the preliminary investigation of this problem at Radiation Laboratory. This method uses a single-wire transmission line located in a shielded room whose metallic walls provide the return circuit for the current, thus effectively forming an "eccentric" transmission line. This single wire is supposedly terminated in its characteristic impedance, and resulting uniform current distribution is assumed. Early work at NRL revealed that this method was unsatisfactory at the frequencies of interest in this problem. Because the effective line spacing is no longer small compared to wavelength, there results a non-uniform field strength which cannot be calculated.

The method described in Reference 14, which uses a loop antenna to develop the radio-frequency field, was next investigated. The formulas for the field strength of this method are developed in Appendix B, based on Reference 14. The equation derived in Reference 14 is not directly applicable for calibration of the loop probe, because of the mutually incompatible requirements that the distance between the loops must be small enough to insure "near-zone" fields, but that the distance also must be much greater than the diameter of either loop.

To insure "near-zone" fields, the maximum spacing between the loops must be less than $\lambda/2\pi$, a distance not much greater than the loop-probe diameter. Since this method is actually based on the mutual inductance between the loops, the correct equation for mutual inductance must be used, rather than the approximation of Reference 14, based on the assumption that the mutual inductance varies inversely with the cube of the distance. The correct equation for mutual inductance between two parallel, coaxial, single-turn circular coils is included in the development given in Appendix B. The resulting formula is rather complex and requires reference to a table (included in Appendix B). That the final formula developed in Appendix B is correct is proven in Table 1. It shows that the loop-probe output voltage varies with spacing in accordance with the formula which includes the true mutual-inductance equation.

TABLE 1

Loop Spacing (cm)	Loop Voltage (Microvolts)		
		Calculated on Basis of Field Strength Indicated by	
	Measured*	$E = \frac{18.85\, I r_1^2 \times 10^6}{X^3}$ † (Eq. 50, Appendix B)	$E = \frac{3IN \times 10^9}{\pi r_2}\sqrt{\frac{r_1}{r_2}}$ † (Eq. 58, Appendix B)
8.5	30.0	44.7	31.2
11.0	16.5	20.5	16.2
14.0	8.5	10.0	8.5
17.0	5.0	5.6	4.9
20.0	3.4	3.4	3.1

* At 2 Mc
† $r_1 = 2$ cm
$r_2 = 4$ cm

Although not explicitly indicated in the formula, the radiating loop must also be balanced. Since most signal generators for this frequency range have an unbalanced output, usually through a coaxial transmission line, the design requirements in respect to balance are the same as for the loop probe.

Since the field strength is directly proportional to the current, the current flowing in the radiating loop must be accurately known. The maximum current which signal generators for this frequency range can develop in the radiating loop is in the order of two milliamperes. This current is limited by the maximum signal-generator output voltage and by the impedance of both the signal generator and the radiating loop. A milliammeter capable of indicating this current yet having a low internal resistance did not appear to be available. The current was measured by adding a 50-ohm non-inductive resistor in series with the loop and measuring the voltage drop across the resistor. A crystal, type 1N34, and a d-c microammeter was used as an r-f voltmeter. Since r-f chokes are not continuously effective at all frequencies, one terminal of the r-f voltmeter must be grounded. Accordingly, the 50-ohm resistor must be connected between one conductor of the loop and ground. This connection requires use of the conventional type of balanced loop (Figure 23 of Appendix A) and prevents use of the simpler type described in Reference 13. A diagram and construction details of the radiating loop assembly and the current-indicating device are shown in Figures 1 and 2.

In the formula developed in Appendix B, it is assumed that the current is of uniform amplitude around the circumference of the loop. This is known to be true only when the circumference of the loop is small compared to wavelength. If the current distribution is uniform, it then follows that the current flowing in the loop is that indicated by the voltage drop across the 50-ohm resistor. In an attempt to achieve uniform current distribution,

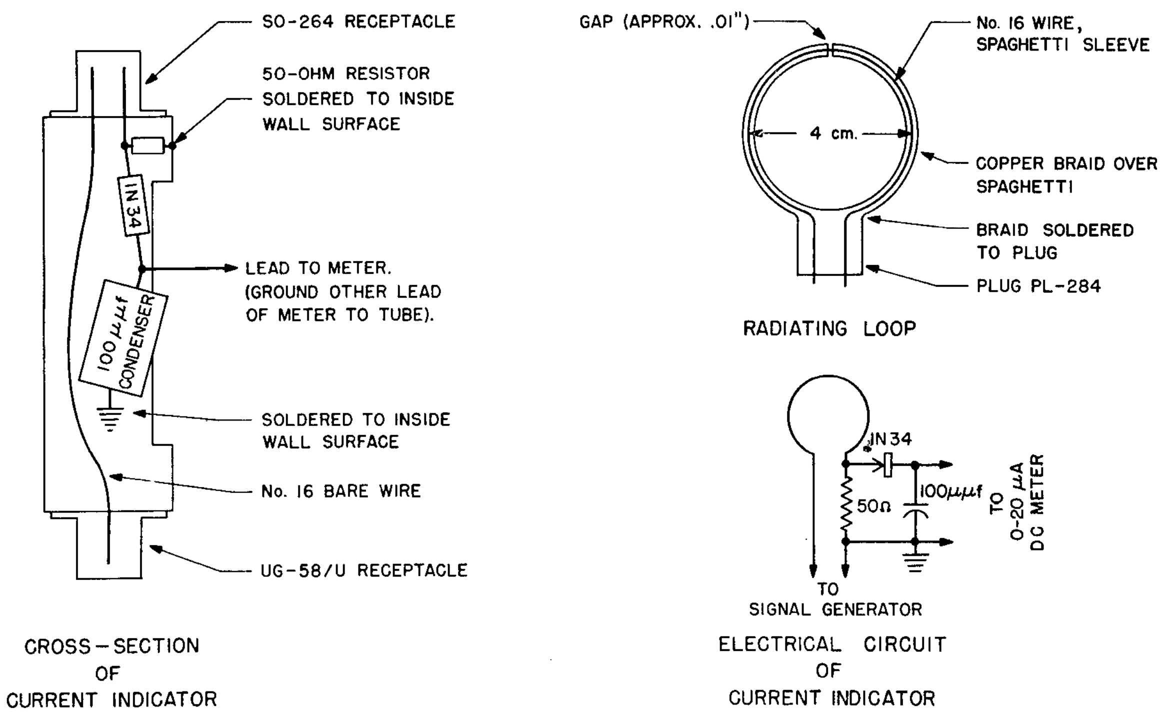


Fig. 1 - Mechanical and electrical details of radiating loop and loop current indicator

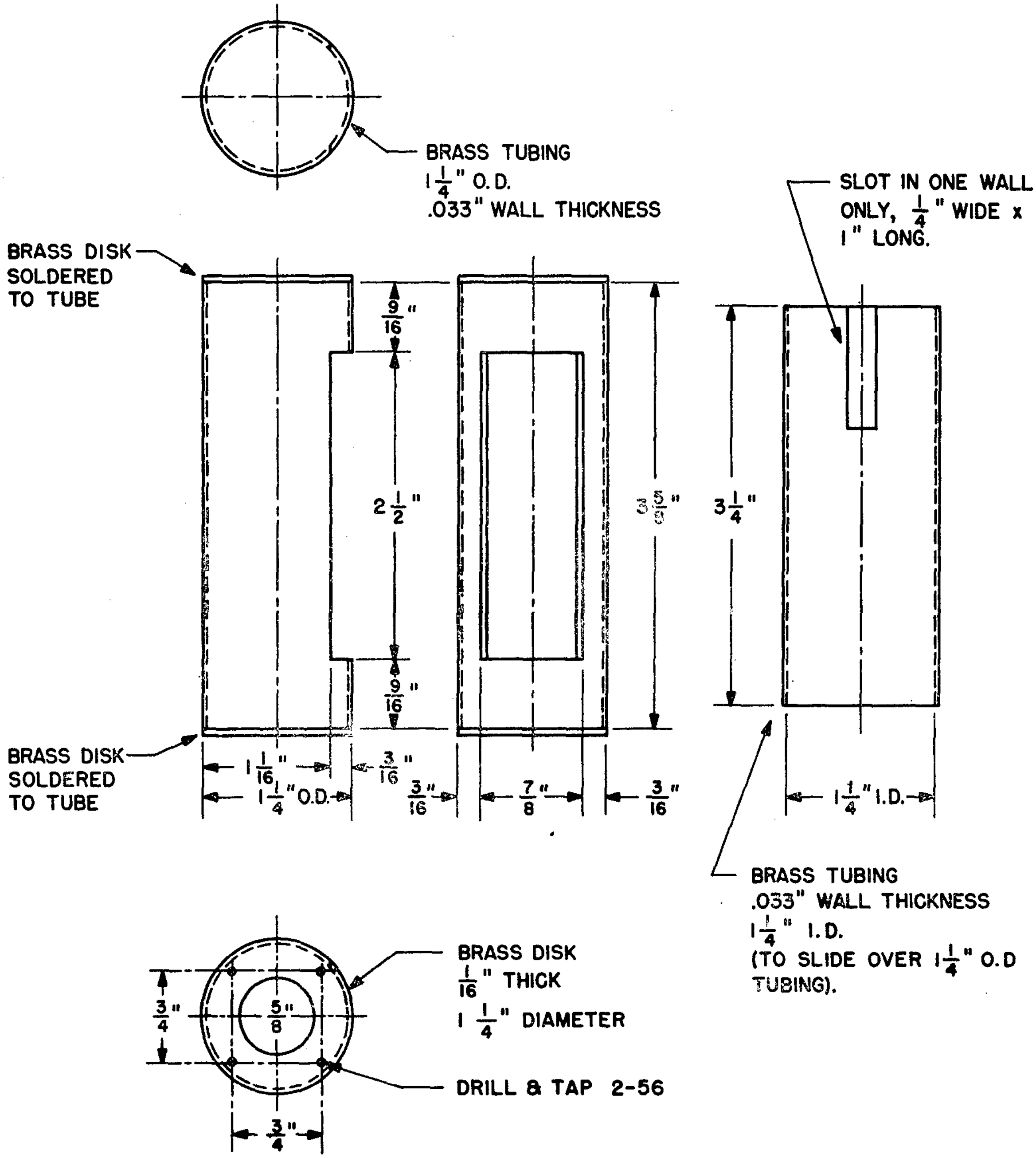


Fig. 2 - Details of tubing for crystal current indicator

the diameter of the radiating loop was made as small as appeared mechanically practicable. However, since there is no way for positively determining the current distribution, experimental proof of the field strength is needed.

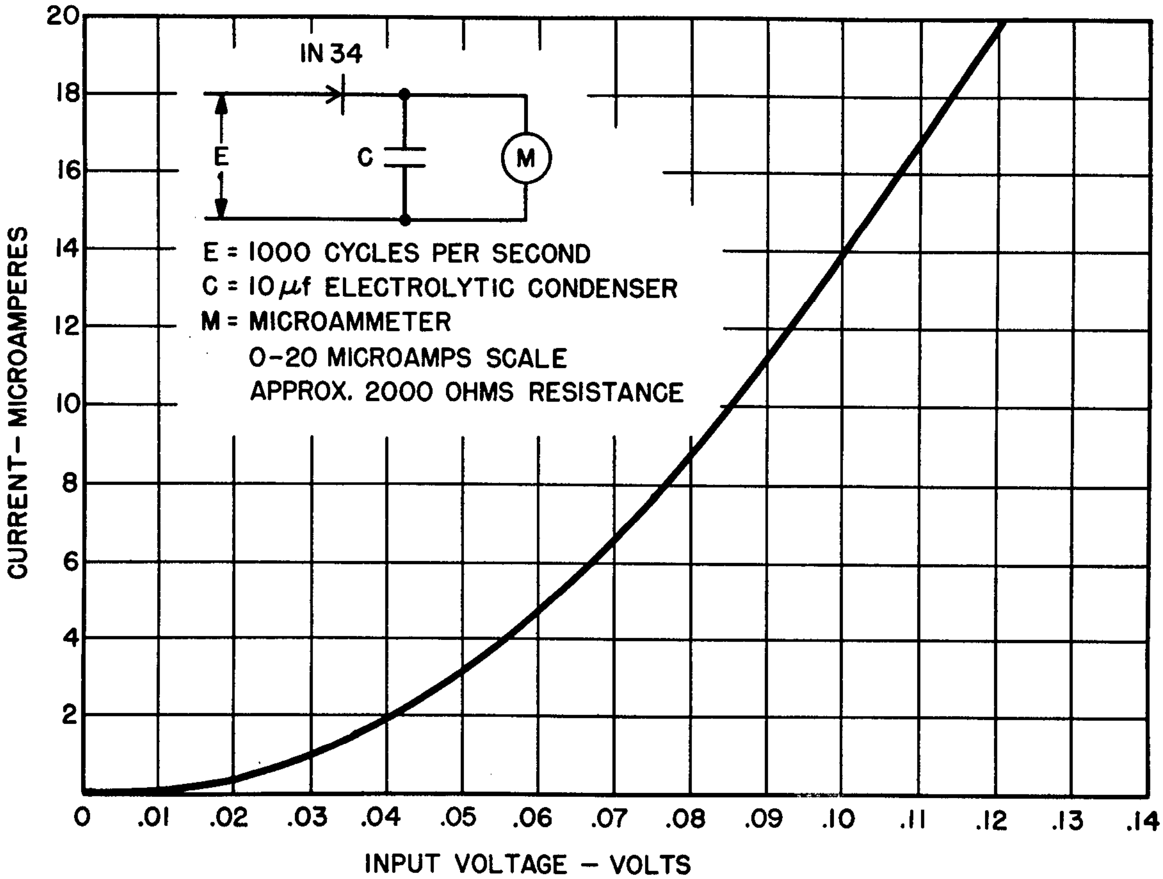


Fig. 3 - Sample calibration of 1N34 crystal used as R-F voltmeter

PROOF OF FIELD STRENGTH

Sensitive, accurate voltmeters are available for audio frequencies, but not for radio frequencies. Therefore, a large capacity condenser was temporarily connected across the d-c microammeter, and the r-f crystal voltmeter was calibrated at a frequency of approximately 1000 cycles. A calibration curve is shown in Figure 3. It is difficult to confirm the accuracy of calibration at radio frequencies. However, the reading of the microammeter was uniform for a constant r-f input at any frequency below 400 Mc. This fact seems to be adequate evidence that the voltmeter calibration is correct for frequencies below 400 Mc. Using a slotted-line, the resistor associated with the voltmeter was measured and found to be a pure resistance of 50 ohms at any frequency below 400 Mc. It was impossible to measure the effect of any stray reactances on the current indicator after assembly in the final form. Care was exerted to keep stray capacity or inductance at a minimum. Such effects are assumed to be negligible.

In order to determine whether the field strength was constant over the desired frequency range, and consequently whether the assumption of substantially uniform current distribution was justified, a simple balanced loop of the type described in Reference 13 was constructed from RG-8/U cable, the same type of cable used to connect the receiver and the loop. This cable was terminated in a resistance equal to its characteristic impedance, and the equivalent circuit was considered to be a simple balanced loop developing voltage across a 50-ohm load.

The impedance of the loop was measured using slotted-line technique and the methods described in Appendix A. A curve of the impedance versus frequency is shown in Figure 4. At frequencies below 400 Mc, the equation of the curve is

$$Z = j\,Z_0 \tan\theta,$$

where Z_0 is a characteristic impedance, and θ is an electrical length expressed in degrees. Calculation for the curve of Figure 4 showed that the characteristic impedance is 183.5 ohms and the electrical length is 0.1648f, where f is the frequency in Mc. These values do not correspond to those calculated by the methods developed in Appendix A or Reference 9. The probable cause of this discrepancy, although not immediately apparent, is the proximity of the ground to the measured loop.

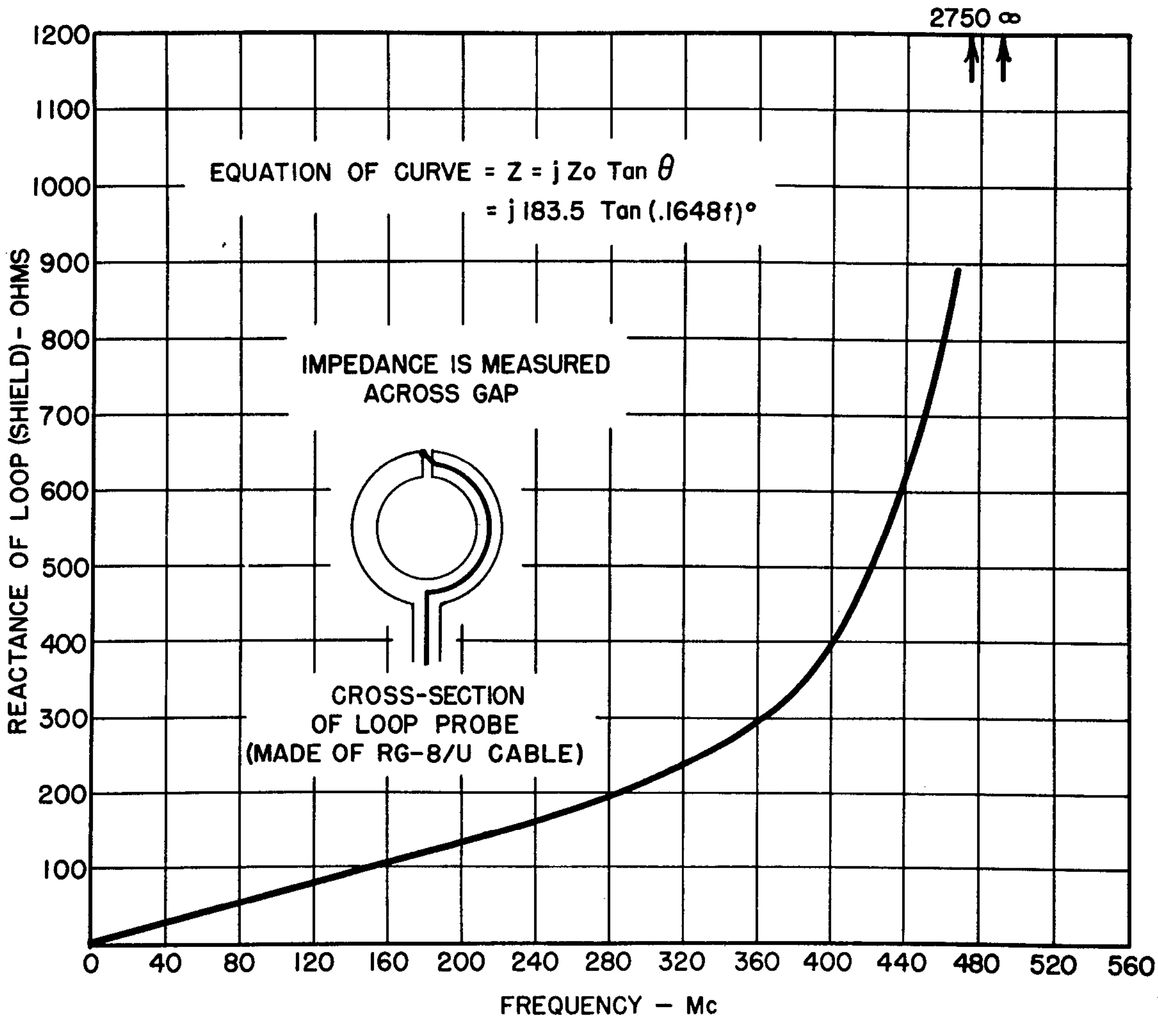


Fig. 4 - Impedance of balanced loop probe made of RG-8/U cable

The simple loop probe, together with the radiating loop, current indicator, and associated equipment, was set up as shown in the block diagram of Figure 5. Using a constant current through the radiating loop, as indicated by the crystal voltmeter, the loop output voltage was measured at intervals of 5 Mc up to 400 Mc. Although the curve of loop output voltage versus frequency showed fluctuations, an average curve was determined as shown in Figure 6. As developed in Appendix A, equation 31, the loop output voltage is

$$e = \frac{hEf}{\cos(kf)^0} \times \frac{R}{\sqrt{R^2 + (Z_0 \tan(kf)^0)^2}},$$

where

h = effective height of loop, meters;
E = field strength, microvolts per meter;
f = frequency, Mc;

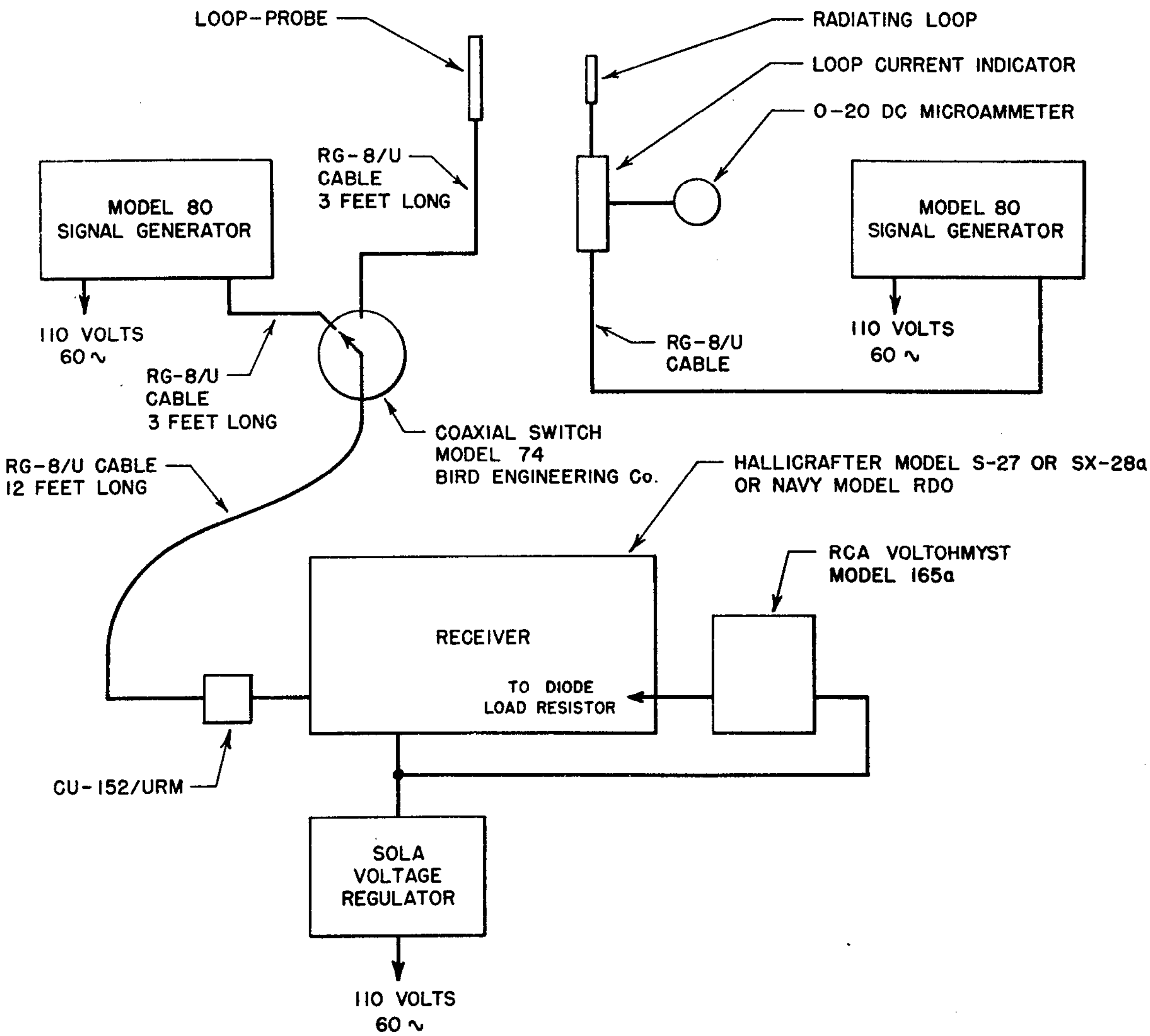


Fig. 5 - Block diagram of setup for calibrating loop probe

R = characteristic impedance of RG-8/U cable;

Z_0 = characteristic impedance of loop considered as a transmission line;

and $(kf)^0$ = electrical length of loop considered as a transmission line.

The value of E, obtained by solving the equation after substituting the known quantities, Z_0, $(kf)^0$ and R, together with calculated values of h and measured values of e, corresponds to that calculated by equation 58 of Appendix B. This agreement appears to be adequate proof of the accuracy of the calculated values of field strength. Moreover, discrepancies between calculated and measured values of loop output voltage during the period of the investigation were always traced to incorrect indication of output voltage by the calibrating signal generator, or to other errors, rather than to incorrect values of field strength.

DETAILS OF CALIBRATING SETUP

In addition to providing the radiating loop and the current-indicating device, mechanical means must also be provided for supporting the radiating loop and the probe to be calibrated in a fixed relation to each other. Since a metal mount may distort the field, the mounts must be made of insulating material.

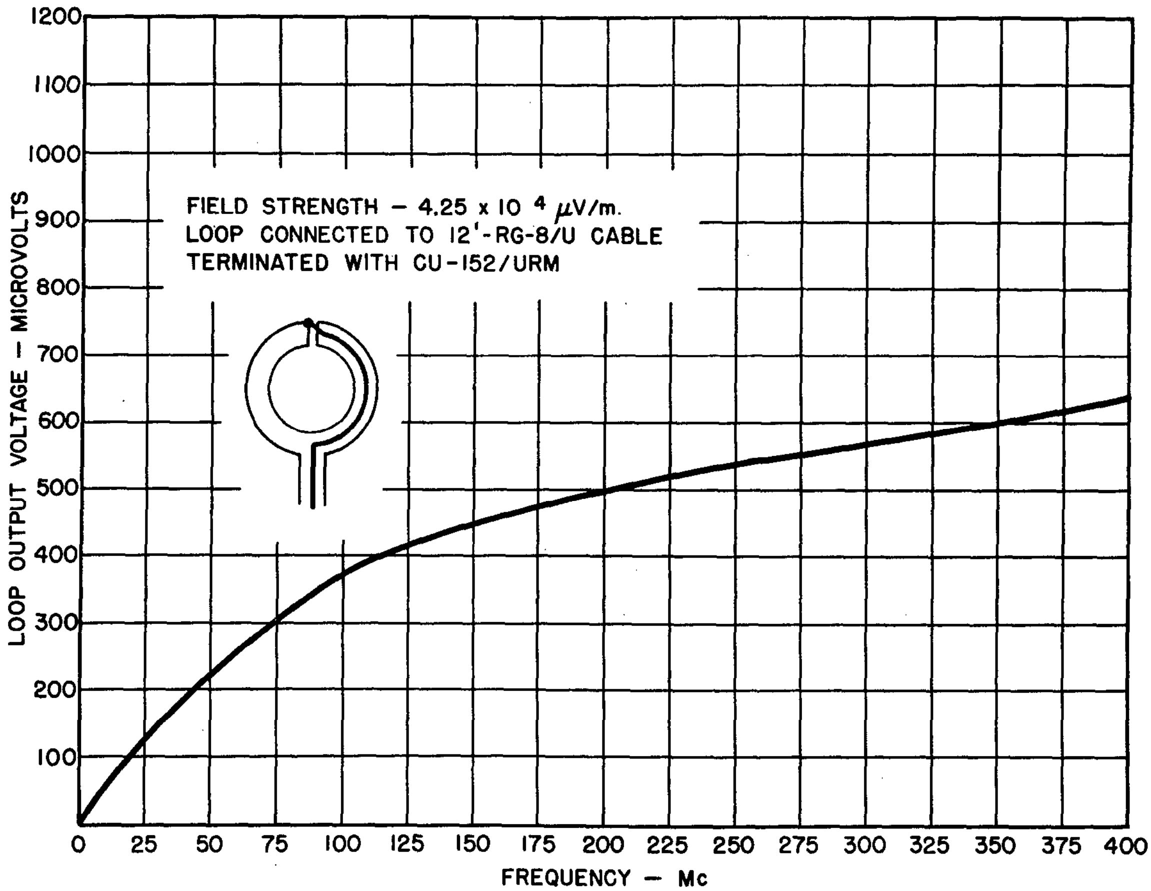


Fig. 6 - Response of balanced loop

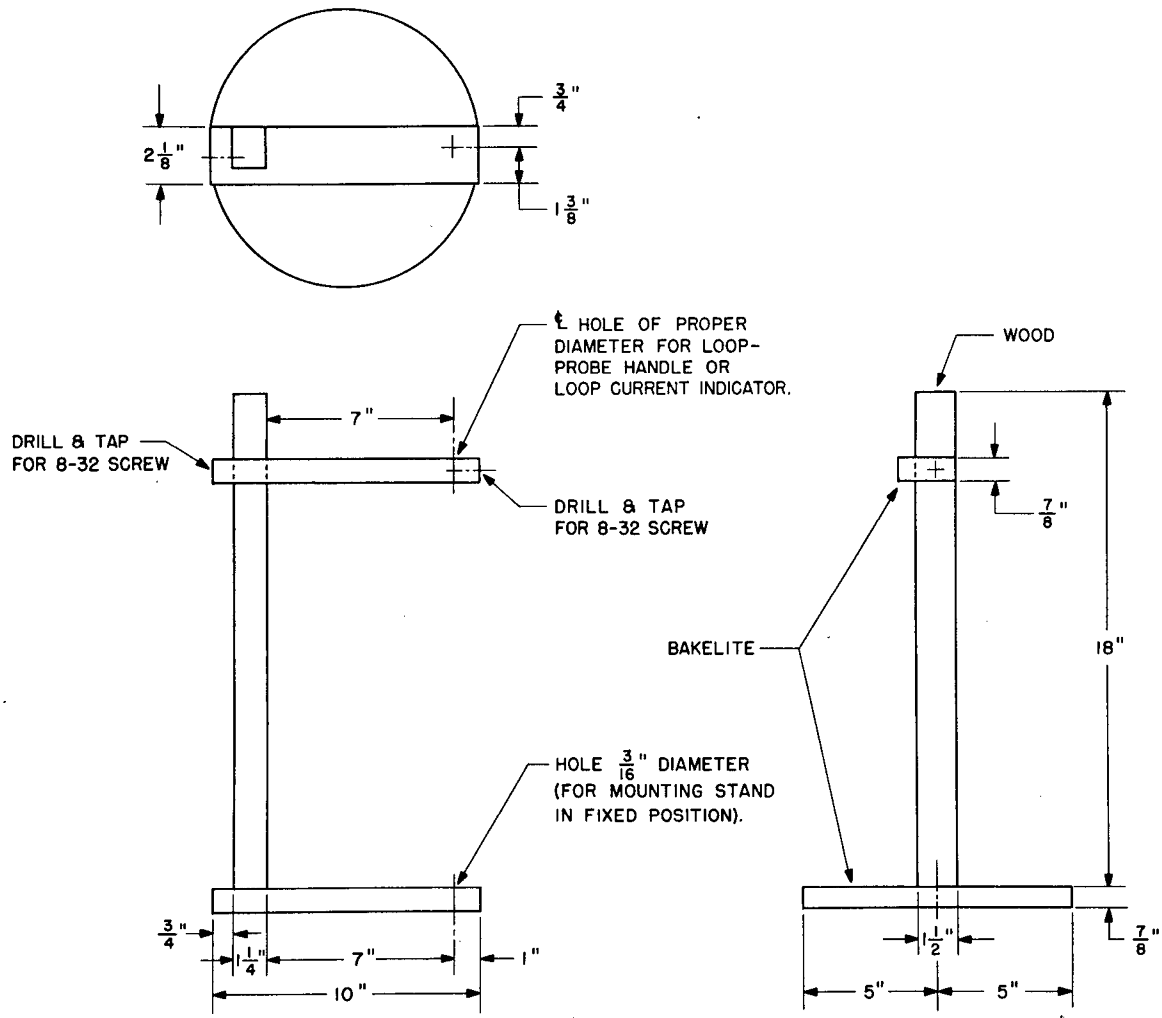
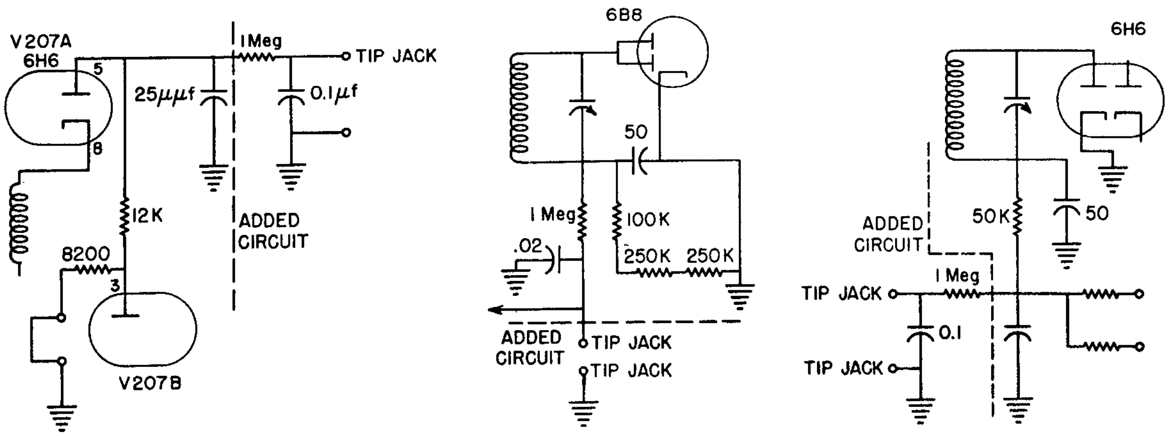


Fig. 7 - Stand for mounting loop probe and radiating loop for loop-probe calibration

Means must be provided for rotating one loop through an angle of at least ninety degrees about its axis of symmetry. If perfect balance is obtained for both loops, if there are no extraneous fields, and if the distance between the loop centers is less than $\lambda/2\pi$, coupling between the loops will be zero when the loop planes are at right angles. Consequently, no voltage will be induced in the loop probe by the radiating loop. The field strength produced by the radiating loop is not known unless these requirements are met. The stands used to mount the loop probes during measurements at the Naval Research Laboratory are shown in Figure 7.

The signal generator or other source of radio-frequency energy used to excite the radiating loop must be very well shielded. Several instruments were checked, and, of those tested, a Measurements Corporation Model 80 was found to be the only commercial signal generator with adequate shielding, frequency range, and power output for the purpose. At some frequencies, the Model 80 signal generator develops satisfactory output power only when the output control is turned much higher than the calibrated points. However, a calibrated signal generator is not necessary for energizing the radiating loop. Any source of radio-frequency energy satisfactory in all other respects may be used without an output calibration.



NAVY RDO HALLICRAFTERS SX-28a HALLICRAFTERS S-27

Fig. 8 - Internal modifications of receivers used for loop-probe calibration

Some form of indicator is of course necessary to measure the loop output voltage. Since this voltage is in the order of microvolts, the indicator must be a sensitive radio receiver which includes some means for indicating the value of the r-f voltage applied to the input terminals. Receivers used at NRL included Hallicrafters Models S-27 and SX-28a, and Navy Model RDO. Each of these receivers includes a signal-strength meter. The response of this meter is approximately logarithmic, however, and the scale is too compressed to permit accurate readings. Therefore, each receiver was modified by adding connections internally so that the d-c voltage developed across the load-resistance of the second-detector diode could be measured. An RCA Model 165 "Voltohmyst" was used as the voltmeter, but any other fairly sensitive, high-resistance d-c voltmeter would be satisfactory. The circuit diagrams of the added receiver connections are shown in Figure 8.

The receivers used as loop-voltage indicators and all transmission lines used in the setup must be well shielded. None of the receivers mentioned was completely satisfactory in this respect. A physical separation of approximately ten feet between the receiver and the radiating loop was necessary in order to prevent false indications due to direct pickup of energy by receiver-circuit elements. An RG-8/U cable approximately twelve feet long was used to connect the loop probe to the receiver input terminals, and the shielding provided by this type of cable was adequate. The receiver antenna-input terminals were modified by installing fittings to connect the coaxial cable to the receiver.

Since the receiver is merely used as an uncalibrated indicator, an accurately calibrated signal generator is required to determine the loop output voltage. The d-c voltage developed at the load resistance of the second detector, due to the loop-voltage, is noted. The loop probe is disconnected from the cable. A signal of the same frequency is applied to the cable by the calibrated signal generator. The signal-generator output voltage is so adjusted that there is developed the same d-c voltage that had been obtained with loop-probe input. The signal-generator output voltage is then equivalent to that developed by the loop probe except that, if necessary, appropriate corrections must be made to include the effect of the signal-generator output impedance.

As shown in Figure 5, a line-voltage regulator was provided for the receiver and voltmeter to improve the stability of the voltmeter indications. The voltmeter readings, however, still showed unexplained fluctuations over a short period of time when a constant r-f voltage was applied to the receiver. The error introduced by the fluctuations may be reduced by rapid determination of the equivalent loop voltage immediately after reading the d-c voltage indication due to the loop-probe voltage. A coaxial-line switch was provided to permit rapid switching of the cable between the loop probe and the calibrated signal generator.

The presence of any large metal surfaces near the loops will distort the field and make results unusable. In particular, the calibration setup will not operate satisfactorily inside a shielded room. There also must be no common connections, such as a ground plane, between the signal generator used to energize the radiating loop and the receiver used as an indicator.

METHOD OF CALIBRATION

The procedure for calibrating a loop probe can now be summarized. The radiating-loop, mounted on the crystal current-indicating device, and the loop probe are mounted in their support mechanisms and located so that (1) the loops are concentric about a common axis and (2) the planes of the loops are parallel. They are spaced a distance apart less than $\lambda/2\pi$, where λ is the wavelength of the highest frequency to be used in calibration. This distance was normally in the order of 10 centimeters, satisfactory for any frequency below 478 Mc. The field strength, for the selected combination of loop dimensions, spacing, and current through the radiating loop, is calculated from the formula of Appendix B.

Before calibration is begun, the setup is checked for balance and for any extraneous fields. This check is made at a high frequency since the effects of improper shielding and unbalance increase with frequency. The radiating loop is energized at approximately the highest frequency to be used in calibration. The receiver connected to the loop probe is tuned to the same frequency, as indicated by a maximum reading of the voltmeter associated with the receiver. One of the loops is now rotated about its own axis of symmetry so that its plane is at right angles to the plane of the other loop. If, under this condition, the receiver indicates absence of voltage output from the probe, the setup is satisfactory with respect to balance, shielding, spacing, and spurious fields.

After the setup is found to be suitable, the loops are returned to the position for calibration, and a known field strength is established by the radiating loop. The reading of the receiver voltmeter is noted, and the loop probe is disconnected from the cable. Replacing the loop probe, a signal generator is connected to the cable and tuned to exactly the same frequency as the receiver. The output voltage of the signal generator is then adjusted until the same reading is obtained on the receiver-voltmeter as that resulting from the loop output. The output voltage indicated by the signal generator is then the same as the loop output voltage, provided the internal impedance of the signal generator can be neglected. If this impedance is not negligible, suitable corrections must be made. Details of the correction depend on the type of signal-generator used. The Model 80 signal generator is calibrated in terms of voltage across a 50-ohm load, and no correction is necessary when an RG-8/U cable, terminated in its characteristic impedance, is used to connect the loop probe to the receiver.

This procedure is repeated, at intervals of approximately five megacycles, over the frequency range for which loop-probe calibration is desired. The calibration factor for the loop probe is then determined at each frequency as the quotient resulting from dividing the field strength by the loop-probe output voltage. A curve of the calibration factor versus frequency is plotted, and a smooth average curve is determined.

DESCRIPTION OF LOOP PROBES

The different types of loop probes studied are shown in Figure 9. There are several variations in details of construction. The single-turn loops submitted by the Signal Corps Engineering Laboratory (Figure 9A) used ceramic beads to support the inner conductor and to space it approximately uniformly in the center of the loop shield. The inner diameters of the "handle" and of the loop shield also differed. Another loop (Figure 9B) submitted by SCEL, had the same dimensions as the single-turn loop of Figure 9A but had six turns forming the loop and did not use ceramic beads as spacers.

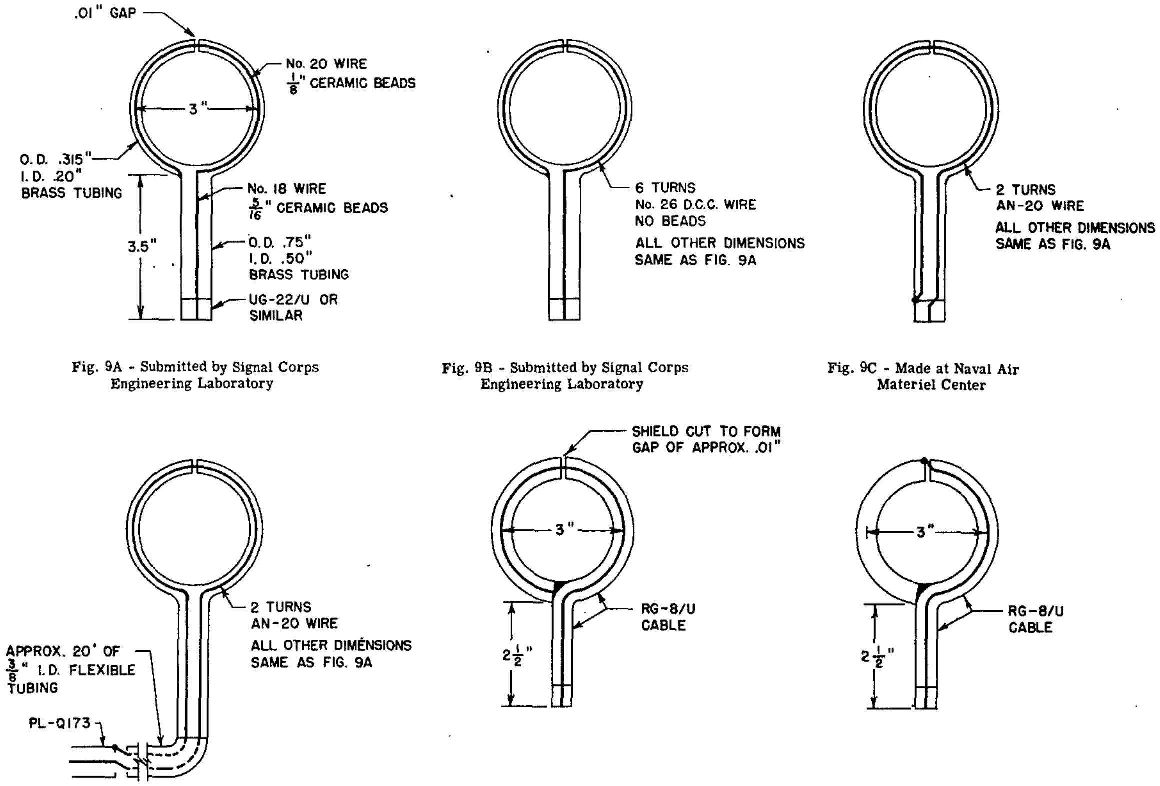


Fig. 9A - Submitted by Signal Corps Engineering Laboratory

Fig. 9B - Submitted by Signal Corps Engineering Laboratory

Fig. 9C - Made at Naval Air Materiel Center

Fig. 9D - Model AN-3065

Fig. 9E - Made at NRL

Fig. 9F - Made at NRL

Fig. 9 - Cross-sectional drawings of specimen loop probes calibrated

The Naval Air Materiel Center loop (Figure 9C) used two turns of insulated wire run loosely through the handle and inside the loop shield but included no means for centering the inner conductors or maintaining them in a fixed position. The AN-3065 loop probe (Figure 9D) differs from all others investigated in that it uses a two-wire transmission line enclosed in a shield to connect the loop to the receiver. The AN-3065 loop is similar internally to the NAMC loop.

Two other types of loops (Figures 9E and 9F) were constructed from RG-8/U cable. These loops have an inner conductor supported by a semi-solid dielectric material and are classed as single-turn loops. At various times during the investigation numerous other loop probes, differing slightly in some detail (such as diameter of tube, size of inner conductor, or length of handle) were constructed, but preliminary investigation did not justify detailed measurements of these modified loops, and no results are reported.

RESULTS OF CALIBRATIONS

Curves of the calibration factor "K" for each type of loop probe are shown in Figures 10 through 15. Figure 16 is a curve of the calibration factor of a three-turn loop not shown in the drawings. Also included is Figure 17 which shows the measured output voltage of each loop, corrected for a standard field of 100,000 microvolts per meter.

As shown in Figure 13, the AN-3065 loop probe, studied first as requested by Reference 1, was calibrated at frequencies below 18 Mc only. This loop has a permanently attached transmission line. The loop terminals are not accessible to permit measurement of the loop voltage. The transmission line consists of two insulated wires twisted loosely together and enclosed, without any supports, in a flexible shield whose inside diameter is much larger than the wire diameter. This cable has no definite characteristic impedance, since there is no fixed geometric relation either between the two wires or between the twisted wires and the shield. The output voltage used to determine values shown in Figure 13 is the voltage developed across the receiver-antenna input terminals, and the curve exhibits the variations with frequency to be expected with an improperly terminated transmission line. This curve cannot be duplicated consistently since bending the flexible shield

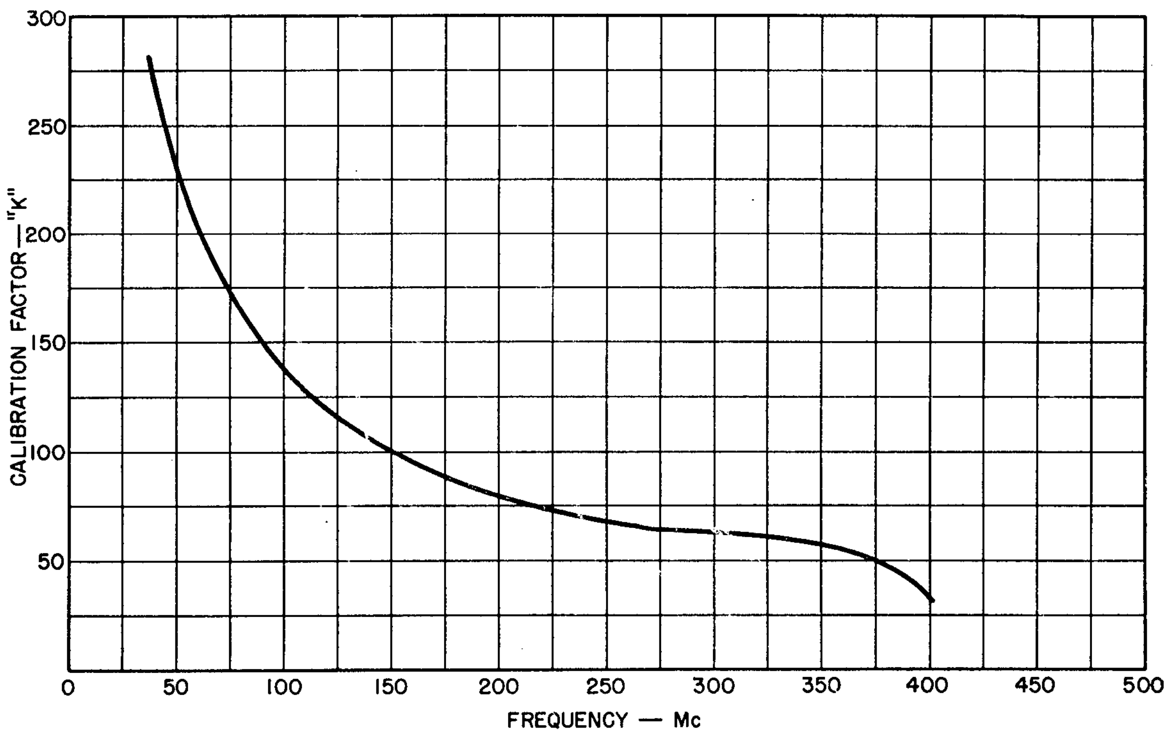


Fig. 10 - Calibration of loop probe shown in Figure 9A

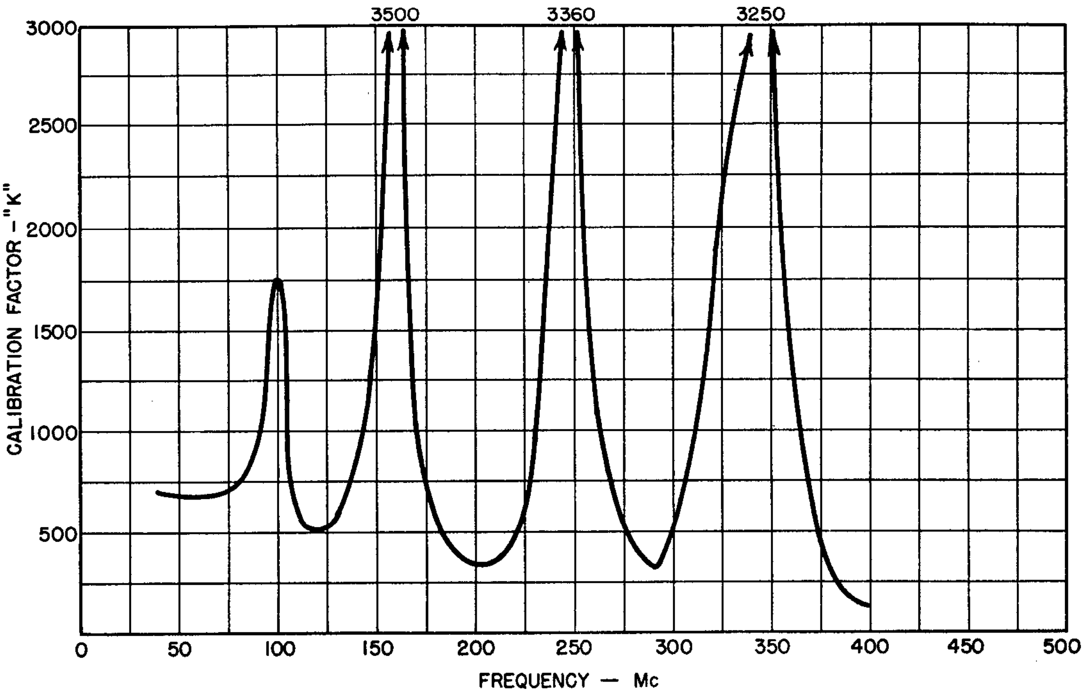


Fig. 11 - Calibration of loop probe shown in Figure 9B

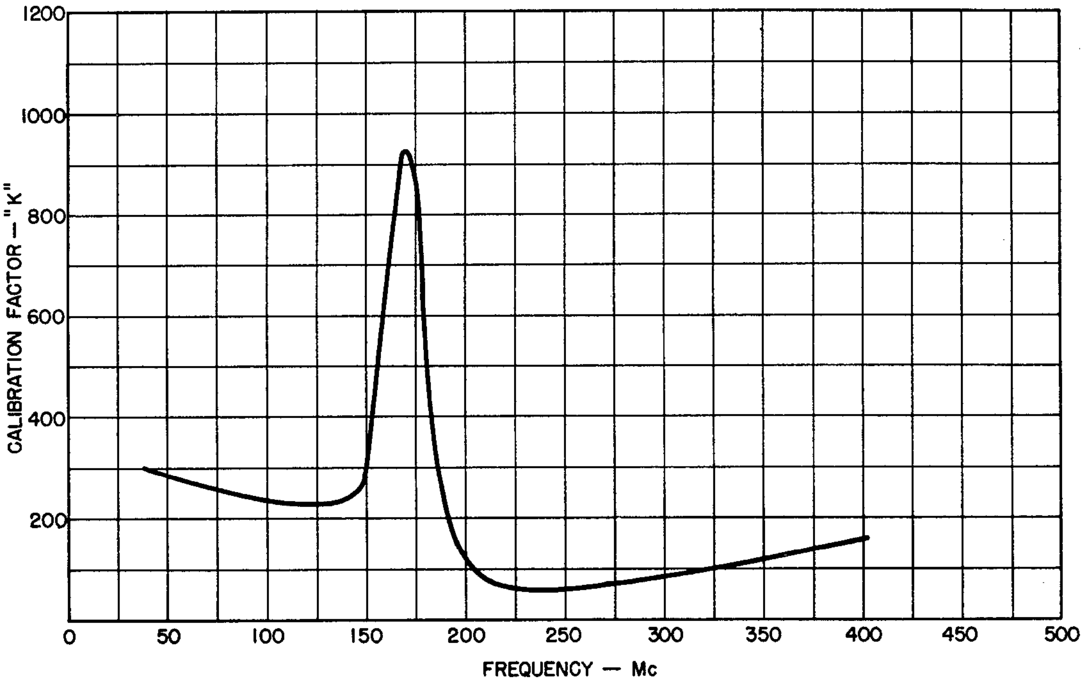


Fig. 12 - Calibration of loop probe shown in Figure 9C

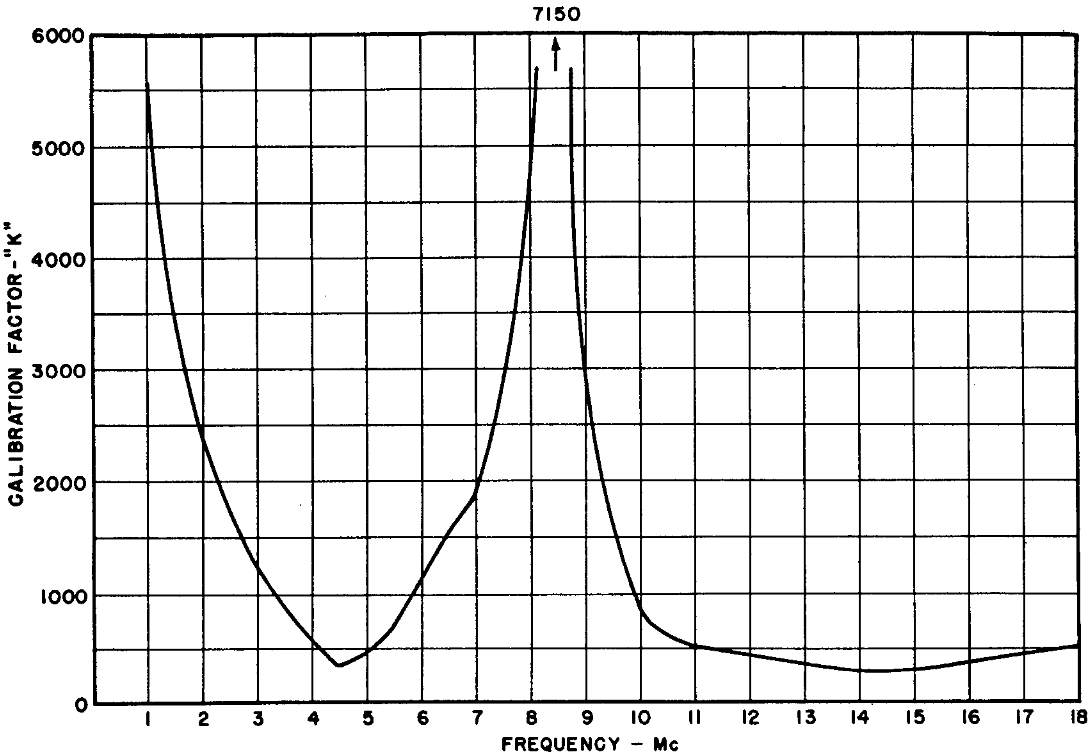


Fig. 13 - Calibration of AN-3065 loop probe (Figure 9D)

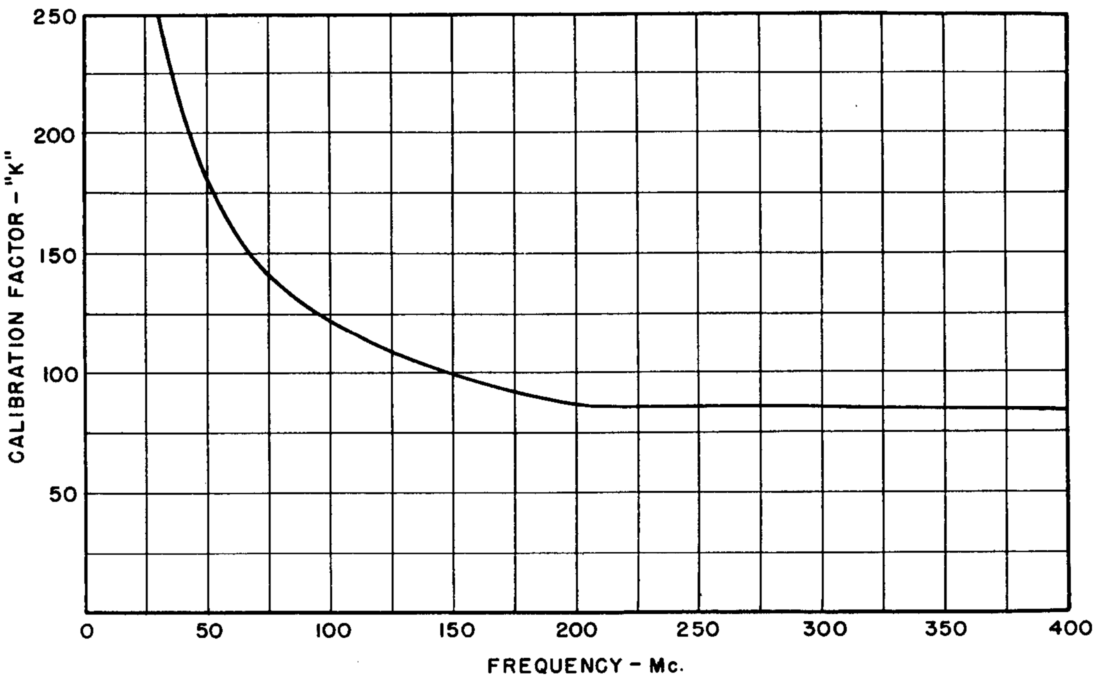


Fig. 14 - Calibration of loop probe shown in Figure 9E

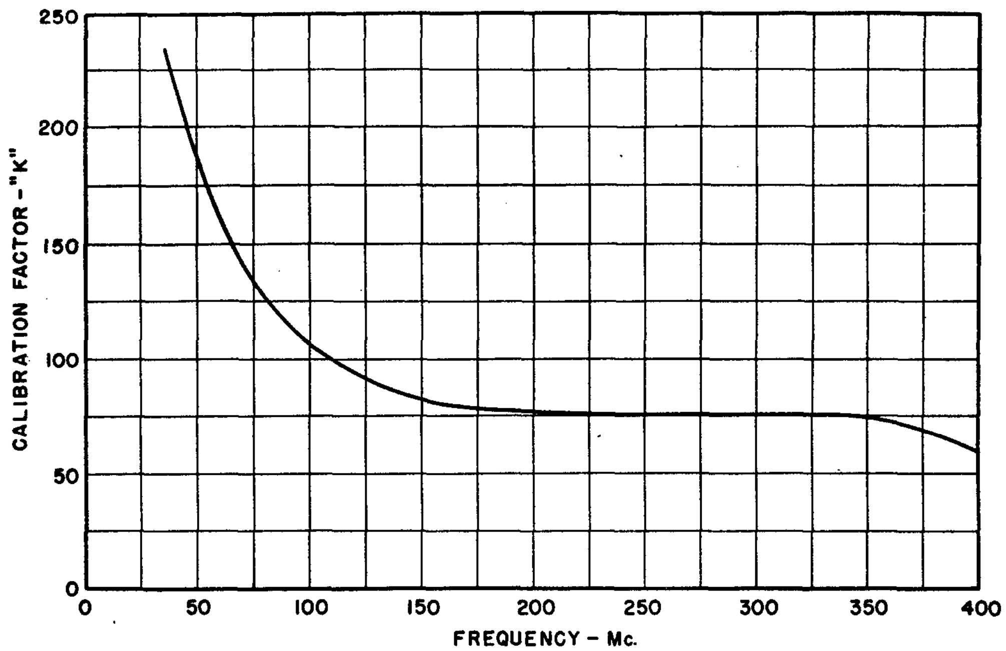


Fig. 15 - Calibration of loop probe shown in Figure 9F

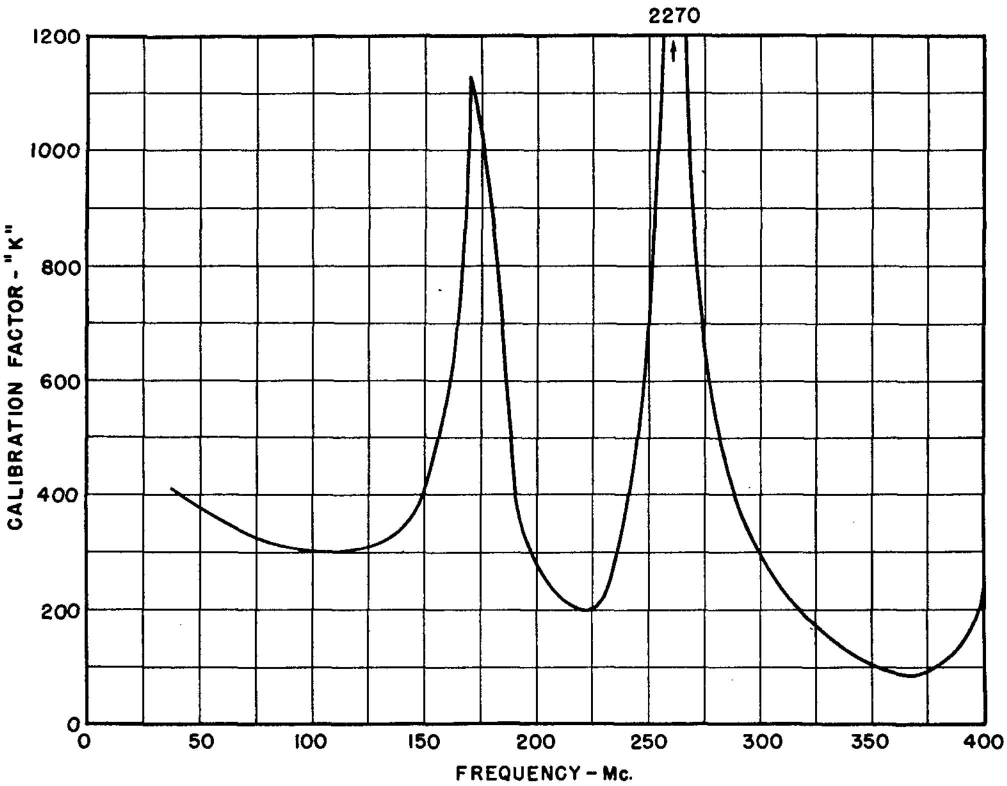
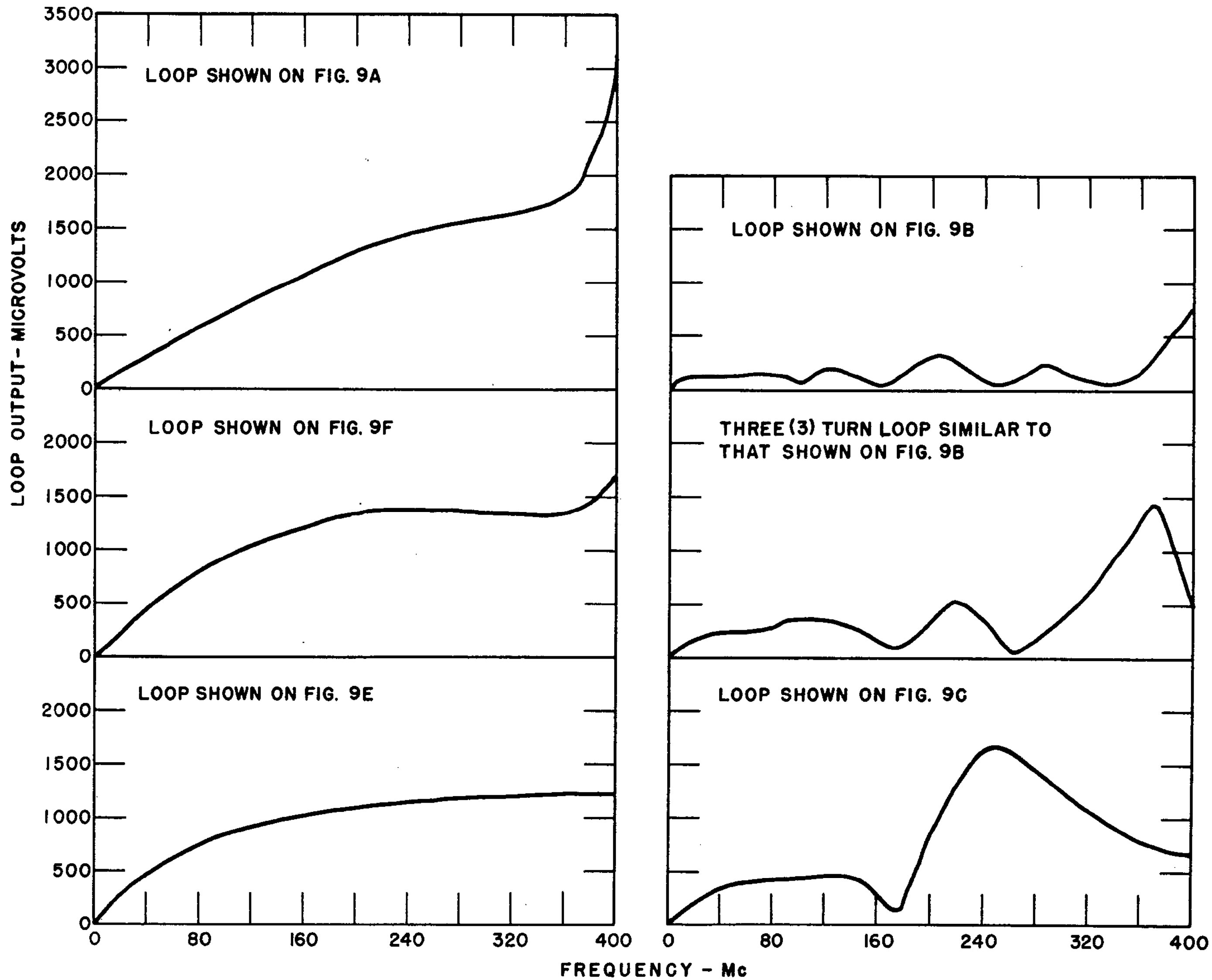


Fig. 16 - Calibration of loop probe similar to that shown in Figure 9B but having three turns



Single-turn loop probes Multi-turn loop probes

Fig. 17 - Output voltage of loop probes in field of 100,000μ v/meter

changes the characteristics of the transmission line, and another calibration will give a different curve. Several attempts were made to provide proper termination for the line by resistance pads, but, as is to be expected in view of the indefinite characteristic impedance, almost no success was attained. This loop was considered unsatisfactory because of the inconsistent calibration and because of the large variations in output voltage. No attempt was made to calibrate it at higher frequencies.

COMMENTS ON LOOP-CALIBRATION RESULTS

It will be noted that single-turn loops had better sensitivity than multi-turn loops at corresponding frequencies above 38 megacycles. Loops of more than one turn also show great variation of response with frequency, while single-turn loops exhibit a response which, within the range of calibration, increases steadily (though not uniformly) with frequency. Of the single-turn loops, the simple balanced loop (as shown in Figure 9F and in Appendix A, Figure 31) appears to have the best combination of sensitivity and uniformity of response.

The better sensitivity of a single-turn loop as compared to a multi-turn loop, when operating into the low load impedance of a terminated transmission line, is explained in Appendix A. Analysis of the equivalent circuit reveals that, at frequencies where the loop diameter is very small compared to wavelength, the output voltage across a low impedance load is directly proportional to the induced voltage and inversely proportional to the sum of the impedance of the loop and the load. The induced voltage is proportional to the area of the loop and to the number of turns. The inductance of the loop is approximately proportional to the square of the number of turns and to the area. Obviously, at a given frequency, the greatest value of induced voltage for the lowest impedance is obtained with a one-turn loop. However, since both induced voltage and loop impedance are proportional to frequency, there is a critical frequency for each number of turns, below which a multi-turn loop will develop more output voltage than a loop having a smaller number of turns. This critical frequency is lower than 38 Mc for any loop having more than one turn of the approximate diameter of the loops studied in this problem.

The points of minimum and maximum response which occurred with multi-turn, balanced, shielded loops can be accounted for by considering each internal coaxial-transmission-line section as having a total length equal to one-half the loop circumference, multiplied by the number of turns, and including this length in the analysis of loop input impedance as developed in Appendix A for single-turn, balanced, shielded loops. It can be assumed that a single-turn loop would exhibit similar deviations from a smooth response curve at higher frequencies, but experimental proof of this has not been established because of the frequency limitations of this method of developing a known radio-frequency field. Even the simple loop shown in Figure 9F will probably display such a characteristic, since the loop itself acts as a transmission line.

EFFECT OF TERMINATING RESISTANCE

The receivers used as voltage indicators do not have an antenna input resistance of 50 ohms at all frequencies. Unless the cable from the loop to the receiver is properly terminated at the receiver by a pure resistance equal to the characteristic impedance of the cable, the load as seen by the loop will vary and will, in addition, contain a reactive component. The curve of calibration factor versus frequency will show points of minimum and maximum values rather than the smooth, steady increase with frequency which should be obtained. But this effect is different from the variations due to multi-turn loops discussed previously. The amount of variation is not as great, while the frequencies of the minimum and maximum values are much closer together.

The amount of error thus introduced depends on the degree of variation of the terminating resistance from the proper value. The error is further increased if the terminating impedance contains a reactance. An attenuator network inserted between the cable and the receiver input terminals will reduce this termination error. Selection of impedance and attenuation ratios for this network requires a compromise since the maximum sensitivity of the loop is also reduced. Enough attenuation must be provided to reduce the effect of the varying receiver input impedance to a negligible value. Yet in order to reduce the loop sensitivity as little as possible, no more than the required minimum attenuation should be used. For the experimental results obtained in this problem, an "L" network was employed. This network, furnished by SCEL and designated CU-152/URM, introduces a minimum attenuation of 20 db. A diagram of this unit is shown in Figure 18. The impedance as seen by the cable should be between 45 and 50 ohms, regardless of the input impedance of the receiver. But it is not this value since the resistance values of the network vary with frequency, and some reactance is also present. Consequently, perfect termination is not achieved. Figure 19 shows a curve of the cable input impedance at the loop end of the cable calculated from measured values of the attenuator impedance. Included are a few measured points.

A pure resistive network providing less attenuation will result in better loop sensitivity, yet should not produce intolerable variations in the cable input impedance. It can readily be shown that the standing-wave ratio on an RG-8/U cable will not exceed 1.5 to 1 if the terminating resistance remains within the limits of 33.3 to 75 ohms. An "L" network designed to provide a 6-db voltage attenuation between a 50-ohm source and a 50-ohm load conforms to this requirement if the receiver input impedance is never less than 10 ohms. Such a pad is also shown in Figure 18.

CONCLUSIONS

A satisfactory method has been developed for calibrating small, shielded, loop probes at frequencies below approximately 400 megacycles. The calibration is in such terms that the probe may be used for repeatable, reasonably accurate measurements of interference fields, with results expressed in absolute terms of microvolts per meter.

A theoretical analysis has been made of the small loop probe operating into a low-impedance coaxial transmission line. Experimental results indicate a lower loop input impedance than predicted by theory, and an explanation for the discrepancy is not apparent.

A simplified loop probe has been studied and found to be the most satisfactory considering uniformity of sensitivity, repeatability and ease of calibration, and mechanical simplicity and ease of construction.

A loop probe approximately three inches in diameter has a maximum usable frequency limit of about 400 megacycles, both because of its response characteristics and because of calibration limitations.

Differences in the characteristic impedance of various sections of the loop probe are the principle sources of variations in the loop response. Improper cable-terminating impedance also produces variations. Both effects can be reduced by proper design.

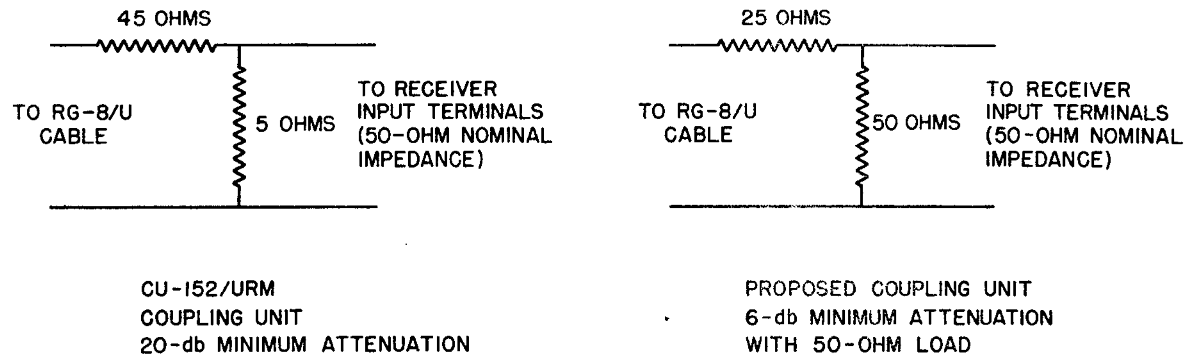


Fig. 18 - Attenuator networks to couple cable to receiver

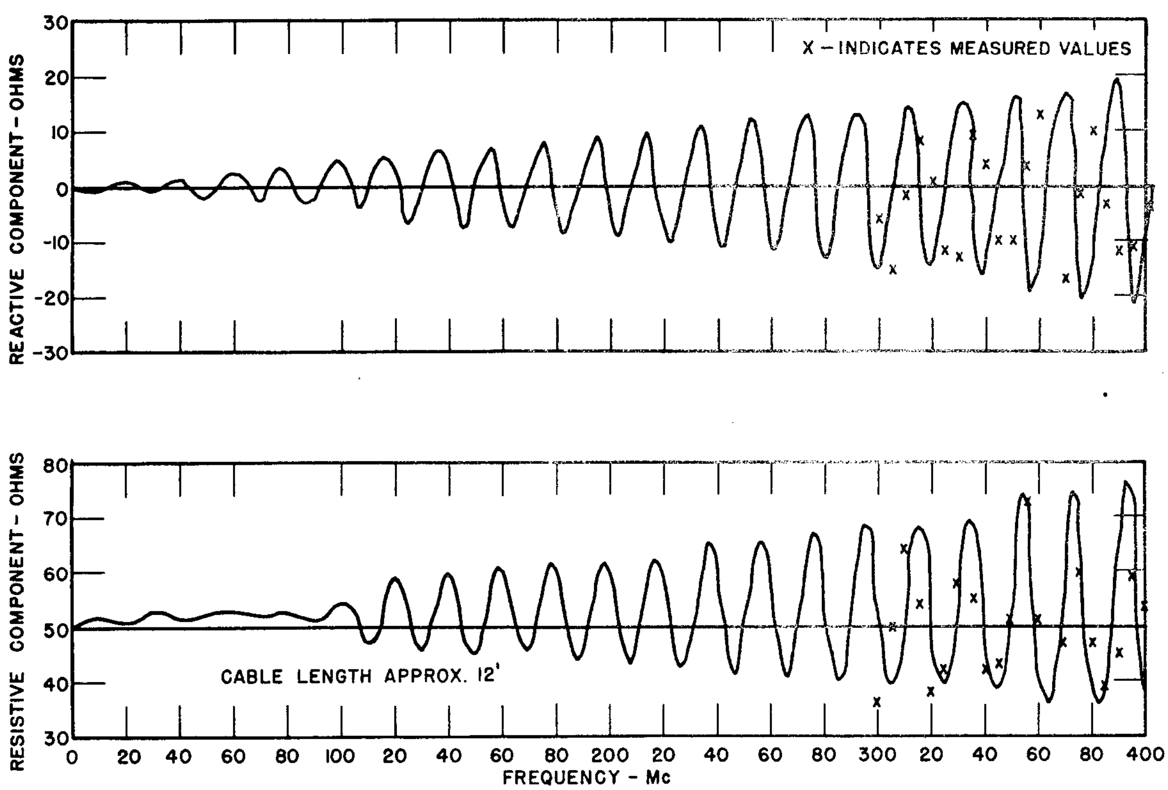


Fig. 19 - Input resistance and reactance of RG-8/U cable terminated with CU-152/URM

RECOMMENDATIONS

The simplified loop probe, of the type shown in Figure 9F, is recommended as the preferable one to be used for measuring radio-interference fields at frequencies between 38 and 400 megacycles. The diameter of the loop should be approximately three inches. The characteristic impedance of the internal section of the loop and the handle should be 52 ohms in order to match the impedance of RG-8/U cable. Velocity of propagation of each section should also be identical.

The method of calibration of loop probes described in this report is recommended for use over the frequency range of 38 to 400 megacycles.

A loop probe is not recommended for absolute measurements at frequencies above 400 megacycles. Therefore, a study is recommended of other types of probe pickup devices for use at higher frequencies.

It is recommended that there be established for radio-interference fields around electronic equipment, limits of maximum permissible field strength which can be correlated with the loop calibration factor and with sensitivity of associated indicating instruments to permit measurements of radio-interference fields on a "go-no go" basis.

* * *

APPENDIX A

EQUATIONS FOR IMPEDANCE AND VOLTAGE OF A LOOP PROBE

At any frequency the voltage developed across a load impedance by a loop antenna located in an electromagnetic field may be calculated from Thevenin's Theorem. The equivalent circuit is shown in Figure 20.

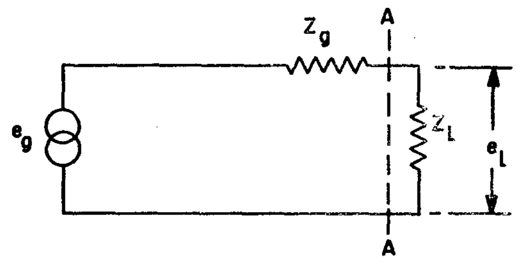


Figure 20

The equation for this circuit is

$$e_L = e_g \frac{Z_L}{Z_g + Z_L}, \tag{1}$$

where A-A indicates the loop terminals,

Z_L = the load impedance,
Z_g = the impedance looking toward the left of A-A, or the impedance which would be measured at terminals A-A when load impedance is disconnected and all sources of voltage are shorted,
e_L = the voltage developed across the load impedance, and
e_g = the voltage developed across terminals A-A when the load impedance is disconnected.

The load impedance is usually known or can readily be measured. The values of e_g and Z_g must be determined by measurement or calculated from the equivalent circuit of the loop.

At low frequencies, where the dimensions of the loop are extremely small compared to the wavelength of the electromagnetic field, the loop capacity, inductance, and resistance can be considered as lumped values, though actually distributed constants. The equivalent circuit for calculating e_g is shown in Figure 21.

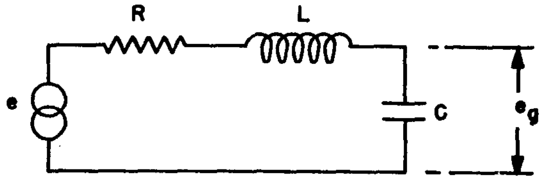


Figure 21

where

R = the total resistance of the loop, ohms;
L = the total inductance of the loop, henries;
C = the total capacity of the loop, farads; and
e = the voltage induced in the loop by the electromagnetic field.

The equation for e_g is

$$e_g = e \frac{X_C}{\sqrt{R^2 + (X_C - X_L)^2}}, \tag{2}$$

where X_L and X_C are reactances of the inductance and capacity at the frequency of interest.

For a rectangular loop, the exact expression for e is*

$$e = 2NbE \sin\left(\frac{\pi a}{\lambda}\right), \tag{3}$$

where N = number of turns in the loop;
λ = wavelength of the electromagnetic field, meters;
E = field strength, volts per meter;
b = height of the loop, meters; and
a = width of the loop, meters.

For a square loop, b = a and equation (3) becomes

$$e = 2NaE \sin\left(\frac{\pi a}{\lambda}\right). \tag{4}$$

For a circular loop, a = average width = $\sqrt{\text{area}} = \sqrt{\pi r^2} = r\sqrt{\pi}$.

If $$a < \frac{\lambda}{6},$$

then $$\sin\left(\frac{\pi a}{\lambda}\right) \cong \frac{\pi a}{\lambda}. \tag{5}$$

---

* It is assumed that the loop is rotated to the position of maximum induced voltage. If this is not true, a term, $\cos\phi$, where ϕ is the angle between the plane of the loop and the direction of travel of the field, must also be included.

On substituting equation (5) in equation (4), for a square loop,

$$e = 2Na\left(\frac{\pi a}{\lambda}\right) E \quad , \tag{6}$$

or, for a circular loop,

$$e = 2Nr\left[\frac{\sqrt{\pi}\left(\pi r \ \sqrt{\pi}\right)}{\lambda}\right] E$$

$$= 2N\,\frac{\pi^2\ r^2}{\lambda}\quad E \tag{7}$$

Since $A = \pi r^2$,

$$e = \frac{2\pi N A E}{\lambda} \ . \tag{8}$$

Since $\lambda = \frac{300}{f}$, where f = frequency of the field, Mc,

$$e = \frac{2\pi f A N E}{300} \ . \tag{9}$$

The equivalent circuit for calculating loop impedance is shown in Figure 22.

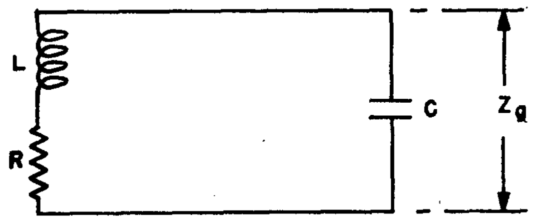


Figure 22

The equation for loop impedance is

$$Z_g = \frac{R + jw\left[L(1 - w^2 LC) - CR^2\right]}{(1 - w^2 LC)^2 + w^2 C^2 R^2} \ , \tag{10}$$

where
Z_g = loop impedance;
L = inductance of the loop, henries;
C = capacity of the loop, farads;
R = resistance of the loop, ohms; and
w = $2\pi f$ where f = frequency, cycles.

For use in direction-finding, the loop antenna is usually tuned to resonance with an external condenser which is effectively in parallel with C. At resonance,

$$X_C = X_L \ ,$$

where X_C = capacitive reactance $= 1/(2\pi f C)$, and
X_L = inductive reactance $= 2\pi f L$.

Equation (2) then becomes

$$e_g = \frac{e X_L}{R} . \tag{11}$$

By definition, $\frac{X_L}{R} = Q.$

Therefore $$e_g = e Q. \tag{12}$$

Loop impedance, at resonance, is

$$Z_g = X_L \sqrt{1 + \frac{X_L^2}{R^2}} = X_L \sqrt{1 + Q^2}. \tag{13}$$

Loop output voltage, across a load impedance, is

$$e_L = e Q \frac{Z_L}{X_L \sqrt{1 + Q^2} + Z_L} . \tag{14}$$

If $$Q \geqq 10, \text{ then } \sqrt{1 + Q^2} \cong Q. \tag{15}$$

Then loop output voltage, across a load impedance, is

$$e_L = \frac{e Q Z_L}{X_L Q + Z_L} . \tag{16}$$

In many cases, $Z_L \gg X_L Q$,

and then $$e_L \cong e Q. \tag{17}$$

It is implicit in all these formulas that the loop is balanced and that the load is also balanced. Unbalance changes both the induced voltage and the loop input impedance in such a manner that calculation of these quantities is impracticable, if not impossible.

The loop probe is, of course, identical with the loop antenna insofar as the loop only is concerned, but the loop probe must be attached to a transmission line. If this transmission line is balanced, the derived equations for a loop operating into a balanced load will still apply. Practical requirements, however, necessitate use of a coaxial transmission line which is unbalanced. Circuit arrangements must be devised which will permit attaching this unbalanced load to the loop without unbalancing the loop. The equivalent circuits for calculating e_g and Z_g must then be derived.

In addition, the foregoing derivations have been based on a circuit composed of lumped constants. At higher frequencies, where loop dimensions approach an appreciable part of a wavelength, the use of lumped values is no longer permissible, and the effect of distributed constants must be taken into account.

Obtaining balanced operation for the loop probe requires use of shielding. Hence, a shielded loop must be evolved and the equations for use of this type of loop with a coaxial transmission line must be derived. To simplify, a single-turn loop is the only form considered.

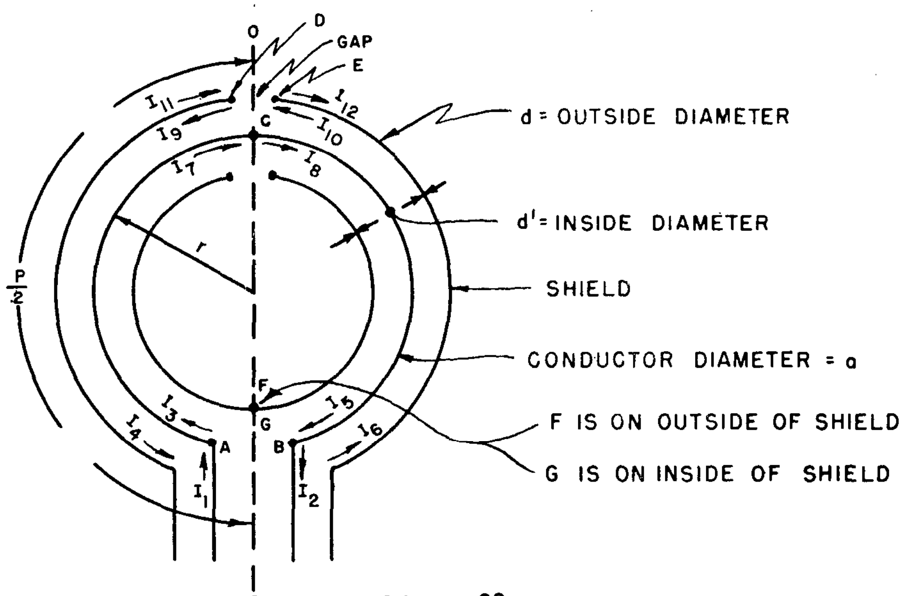


Figure 23

Figure 23 is a cross-sectional drawing of a balanced, shielded, single-turn loop connected to a two-wire, shielded, balanced transmission line. The following assumptions are made:

1. Shield is of high-conductivity metal, such as copper or aluminum.
2. Shield thickness is much greater than depth of penetration so that current on outside shield surface does not penetrate through the shield.
3. Loop is physically and electrically symmetrical about axis 0-0.
4. Width of gap is very small but capacity across gap is negligible.
5. Series resistance and shunt conductance are negligible.

The equivalent circuit can be derived by an analysis of the instantaneous current flow.

The letters A, B, C, D, etc. shown on the drawing designate the points shown as dots. The arrows associated with the various currents indicate the instantaneous direction of flow of current at the designated points.

A number of current relationships may be shown to exist. First,

$$I_1 = I_2,$$

based on the assumed condition of a balanced transmission line. There is no current flowing on the inside surface of the shield around a balanced line.

Second, the inner conductor from A to C, together with the inside surfaces of the shield, obviously forms a coaxial transmission line. The same is true from B to C. Since this is true,

$$I_3 = I_4,$$

where I_4 is the current on the inner surface of the shield. This current, of course, is distributed around the entire inner circumference of the shield.

Also

$$I_5 = I_6,$$

for the same reason.

Next

$$I_7 = I_8.$$

Since C is a point, obviously the current entering the point is the same as that leaving the point.

Then

$$I_7 = I_9$$

for the same reasons that $I_3 = I_4$.

Further,

$$I_{11} = I_9.$$

I_{11} is the current flowing on the outside surface of the shield, while I_9 is that on the inside surface of the shield. Since D is a point on the gap in the shielding, obviously current entering a point is equal to current leaving the same point.

Then,

$$I_{10} = I_{12}$$

for the same reasons that $I_{11} = I_9$.

Since

$$I_9 = I_7,$$

$$I_7 = I_8, \quad \text{and}$$

$$I_8 = I_{10},$$

then

$$I_9 = I_{10}.$$

Also

$$I_{11} = I_{12},$$

since

$$I_{11} = I_9,$$

and

$$I_9 = I_{10}.$$

From this analysis, it is apparent that the current flowing on the outside shield surface is unity-coupled, at the gap, to the inside surface, resulting in current flow on the coaxial

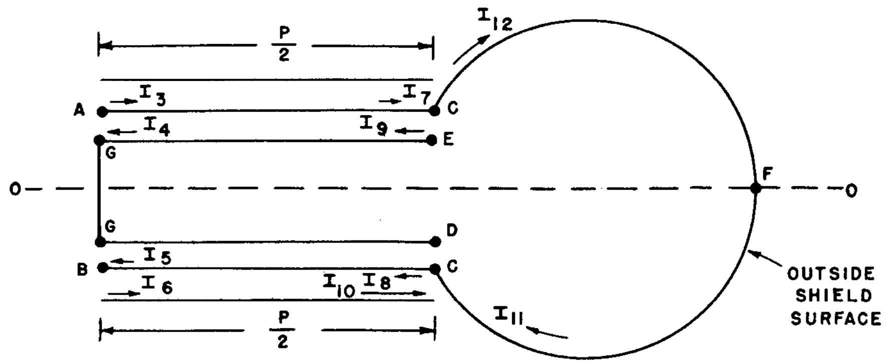


Figure 24

transmission lines. The latter, in turn, produces current flow on the balanced transmission line used to connect the loop to a load at some distant point. The equivalent circuit, considering points A and B as the "loop" terminals, is shown in Figure 24. Input impedance at terminals A-B is that of the "twin-coaxial" type of balanced transmission line, terminated in the impedance of a loop formed by the outside shield surfaces.

A cross-sectional drawing of a balanced shielded loop connected to a coaxial transmission line is shown in Figure 25. Here the equivalent circuit is the same as that of the balanced loop operating into a balanced transmission line, as shown in Figure 24, except that a short is connected from B to G. The equivalent circuit is shown in Figure 26.

The input impedance at terminals A-G is that of a coaxial line terminated in a load impedance. The load impedance consists of the loop-shield impedance in series with the input impedance of another section of coaxial line shorted at B-G.

It is apparent that the transmission line section between D and G merely increases the impedance of the loop, as seen from the loop terminals A-G. Since the transmission line is shielded, it can contribute nothing toward the voltage developed at A-G. A lower loop impedance will result if the extra section is eliminated, as shown in Figure 27.

Since the section of line from C to B is shorted at both ends, the center lead, C-B, is effectively removed, and the equivalent circuit becomes that shown in Figure 28. This loop differs from a simple open loop at the end of a coaxial line in that it is balanced.

Evaluation of the input impedance of this loop is simple, since it is merely a section of coaxial line terminated in the loop-shield impedance. At low frequencies, the loop shield may be considered as a pure inductance. The input impedance at terminals A-G is then

$$Z = j\, Z_o \tan\left(\theta + \arctan \frac{X_L}{Z_o}\right), \tag{18}$$

where Z_o = the characteristic impedance of the coaxial line,
θ = the electrical length of the coaxial line, and
X_L = the inductive reactance of the loop.

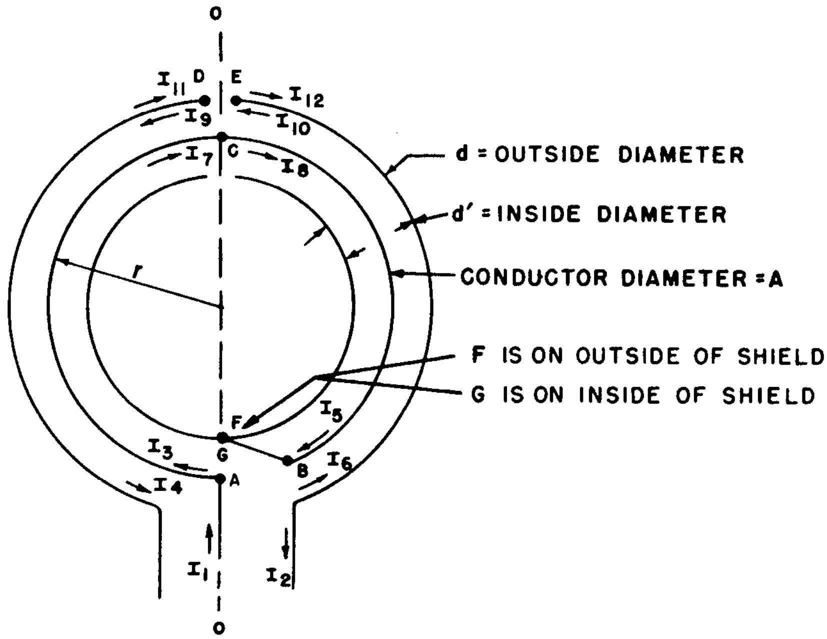


Figure 25

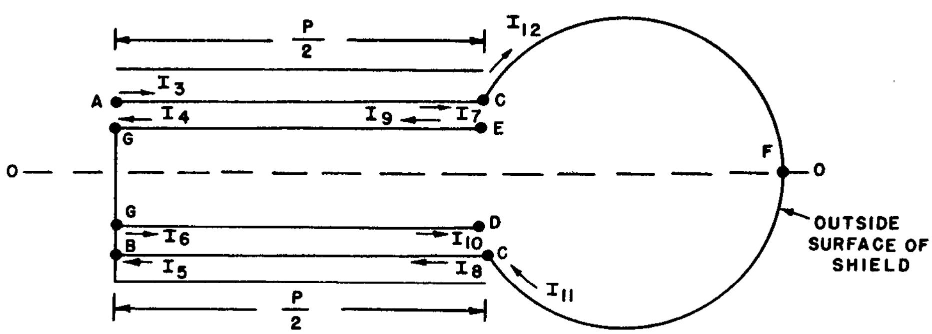


Figure 26

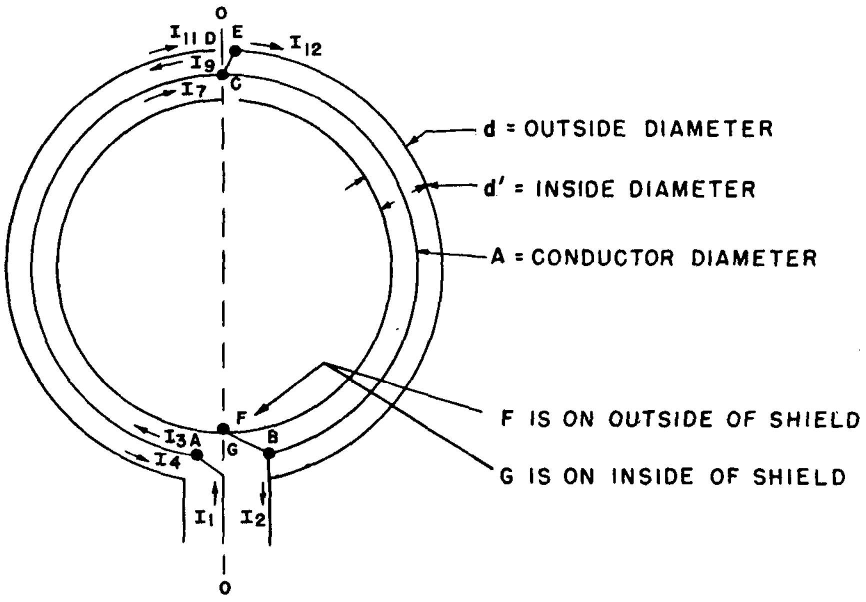


Figure 27

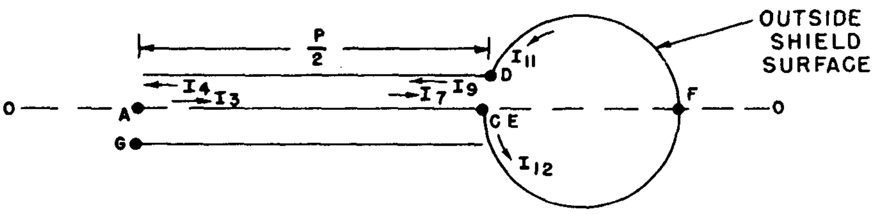


Figure 28

From basic transmission-line equations,

$$Z_0 = 138 \log_{10} \frac{d'}{a} , \tag{19}$$

where d′ and a are dimensions shown in Figure 27. The electrical length is calculated from

$$\theta = \frac{2\pi\left(\frac{p}{2}\right)}{\lambda} \quad \text{or} \quad \frac{2\pi\left(\frac{p}{2}\right) f}{300} , \tag{20}$$

where $\frac{p}{2} = \pi$ r, and

r = radius of the loop.

Inductive reactance is calculated from

$$X_L = 2\pi f L, \tag{21}$$

where f = frequency in Mc, and
L = inductance of loop in microhenries.

At higher frequencies, the loop shield can no longer be considered as a pure inductance but must be considered as a nonuniform transmission line. Redrawing the loop only, as shown in Figure 29, the following simplifying assumptions are made to determine the equivalent circuit:

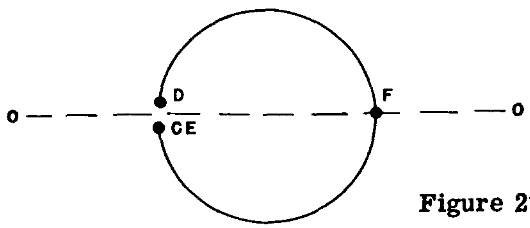


Figure 29

1. The loop is a nonuniform transmission line, shorted at point F;

2. The loop is equivalent to a uniform, two-wire, balanced transmission line having the same area as the loop and having a length equal to half the loop circumference (the equivalent line spacing is then obtained by dividing the area of the loop by the equivalent length. This, in the case of a circular loop, reduces to the radius of the loop, r.);

3. The diameter of the conductors of the equivalent transmission line is the same as the outer diameter of the loop shield, d;

4. The characteristic impedance may then be calculated from

$$Z_o = 120 \cosh^{-1} \frac{r}{d} \tag{22}$$

$$\cong 276 \log_{10} \frac{2r}{d} \;; \tag{23}$$

5. The input impedance may then be calculated from the formula for the input impedance of a shorted transmission line,

$$Z = j Z_o \tan \theta , \tag{24}$$

where Z_o is the characteristic impedance calculated from equation (22) or (23), and θ is the equivalent electrical length of the line, which is calculated from

$$\theta \text{ (radians)} = \frac{2\pi \left(\frac{p}{2}\right)}{\lambda} = \frac{2\pi \left(\frac{p}{2}\right)}{300} \tag{25}$$

or

$$\theta \text{ (degrees)} = \frac{360^\circ \left(\frac{p}{2}\right)}{\lambda} = \frac{360^\circ \left(\frac{p}{2}\right) f}{300} , \tag{26}$$

where

p = perimeter of the loop in meters,
λ = wavelength in meters, and
f = frequency in Mc.

All the equivalent circuits developed have been circuits for calculating the impedance at the loop terminals. However, all loop probes include, as a "handle," another length of coaxial transmission line connecting the loop terminals to a plug which is used to connect the probe to a coaxial cable. The equivalent circuit for the type of loop shown in Figure 25, with the added cable forming the "handle," is shown in Figure 30.

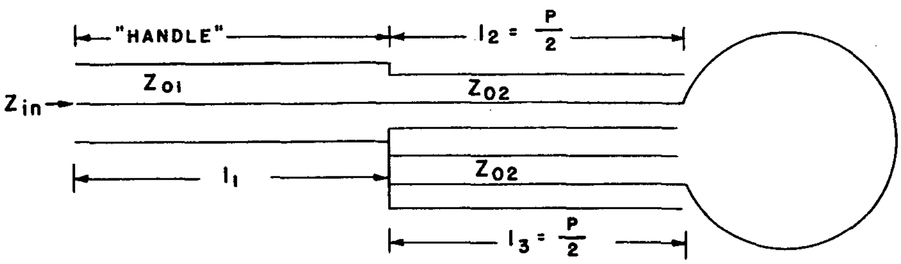


Figure 30

Designating the loop impedance as Z_I, the input impedance is

$$Z_{in} = j Z_{01} \tan \left(\theta_1 + \arctan \left[\frac{Z_{02} \tan\left(\theta_2 + \arctan \left(\frac{Z_I + Z_{02} \tan\theta_3}{Z_{02}} \right)\right)}{Z_{01}} \right] \right), \tag{27}$$

where Z_{01} and Z_{02} = characteristic impedances of the coaxial line sections shown in Figure 30, and

θ_1, θ_2, and θ_3 = electrical lengths of the sections.

Figure 31 shows the equivalent circuit of the type of loop shown in Figure 27, with the added "handle."

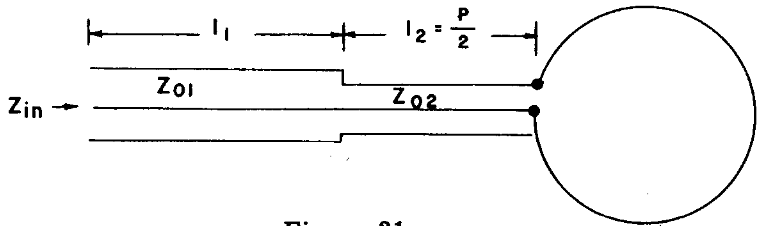


Figure 31

The input impedance of this loop is calculated from

$$Z_{in} = j Z_{01} \tan\left[\theta_1 + \arctan \frac{Z_{02} \tan\left(\theta_2 + \arctan \frac{Z_I}{Z_2}\right)}{Z_{01}}\right], \tag{28}$$

where the symbols have the same meaning as defined for equation (27).

It is apparent that either loop would be simplified if the characteristic impedance of the "handle" section, Z_{01}, were the same as the characteristic impedance of the "internal" coaxial transmission-line section, Z_{02}. In the case of the simpler loop shown in Figure 31, if $Z_{01} = Z_{02}$, and if the characteristic impedance of the coaxial cable used to connect the loop probe to the indicator is also equal to Z_{01}, the equivalent loop circuit will become that of a simple loop on the end of a coaxial transmission line of characteristic impedance Z_{01}. If the cable is terminated, at the receiver-indicator in a resistance $R = Z_{01}$, the equivalent circuit becomes simply a loop connected to a resistor as shown in Figure 32.

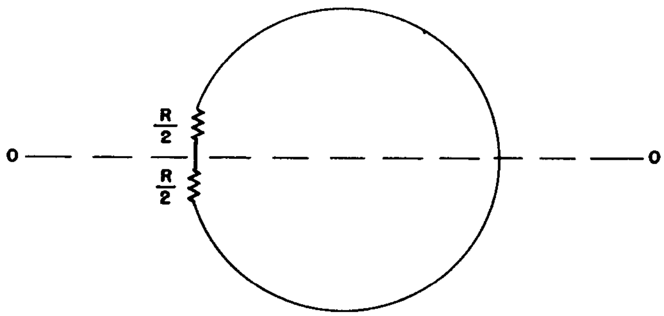


Figure 32

When the loop is considered as an equivalent uniform transmission line, the equivalent circuit is as shown in Figure 33.

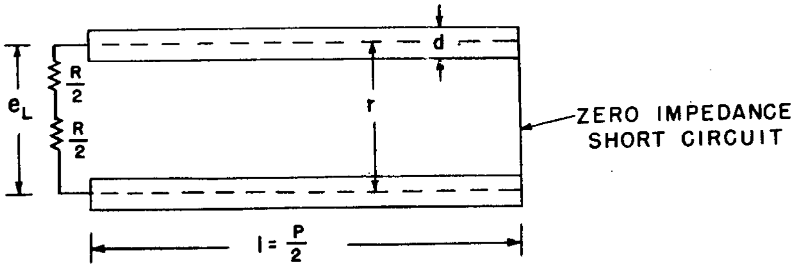


Figure 33

The equivalent characteristic impedance, designated $Z_{0_{eq}}$, is calculated from equation (22) or (23). The equivalent loop impedance, designated Z_I, is calculated from equation (24). By substituting Z_I for Z_g and R for Z_L in equation (1), the following equation for the loop output voltage is obtained.

$$e_L = e_g \frac{R}{\sqrt{R^2 + Z_I^2}} = e_g \frac{R}{\sqrt{R^2 + \left(Z_{0_{eq}} \tan\theta\right)^2}} . \tag{29}$$

It is further assumed that the induced voltage, e, can be considered as a lumped voltage located in the zero-impedance short circuit, as shown in Figure 34.

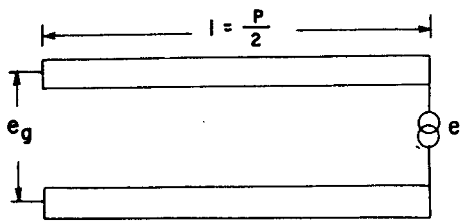


Figure 34

Since the loop is considered equivalent to a transmission line in impedance, the open-circuit voltage at the loop terminals must also conform to that of a transmission line. Voltage e_g therefore is inversely proportional to the cosine of the equivalent length of the loop, as shown in the equation,

$$e_g = \frac{e}{\cos\theta} , \tag{30}$$

where
e_g = the open-circuit voltage of the loop,
e = the voltage induced in the loop by the electromagnetic field, and
θ = the equivalent electrical length of the loop.

The complete equation for the loop output voltage developed across a load resistor is

$$e_L = \left(\frac{e}{\cos\theta}\right)\left(\frac{R}{\sqrt{R^2 + (Z_{0_{eq}} \tan\theta)^2}}\right) , \tag{31}$$

where e is calculated from equation (9).

If $$h = \frac{2\pi A N}{300},$$

equation (9) becomes $$e = h E f. \qquad (32)$$

Equation 31 is based on three assumptions, namely:

1. That the loop can be considered as equivalent to a balanced, uniform transmission line;

2. That the voltage induced in a loop by an electromagnetic field is, at all frequencies of interest in this problem, proportional to the frequency, to the area and number of turns of the loop, and to the field strength; and

3. That the induced voltage can be considered as lumped in the zero impedance short-circuit of the equivalent transmission line.

Experimental proof of the correctness of these assumptions is necessary. The first assumption appears to offer the greatest possibility of proof, since impedance in the desired frequency range can be readily measured by use of transmission line techniques. There are,however, limitations to this measurement which should be discussed.

The input connection on the "handle" is the only point of a loop probe accessible for measurements. Connection to any other point on the probe will upset the balance of the loop and render measurements useless. Input impedance must be measured at the handle for a number of frequencies. The measured values of input impedance, together with corresponding frequencies, are then used to form simultaneous equations, which are solved to determine the unknown electrical lengths and characteristic impedances.

Measurement of the loop shown in Figure 25 is impractical. Study of equation (27) shows that there are three unknown characteristic impedances and three unknown electrical angles involved. These variables cannot be separated by measurements at the loop-probe handle. It may appear that the characteristic impedance and the electrical length of the various sections of coaxial transmission line can be calculated from the dimensions. The center conductors of these sections of transmission line are supported by some form of insulation, such as ceramic beads, which introduces an unknown dielectric constant into the formula for characteristic impedance and which also affects the velocity of propagation and consequently the electrical length. Calculations which include all these factors as approximations do not yield results of the desired accuracy.

Even the simplified loop shown as an equivalent circuit in Figure 31 cannot be measured until it is further simplified in construction by making the characteristic impedance $Z_{01} = Z_{02}$ and making the velocity of propagation identical in both sections. Under this condition the input impedance is simply that of a coaxial transmission line with a reactance connected at the far end. If the coaxial-line characteristic impedance and velocity of propagation are known, the loop impedance may readily be calculated.

If the line characteristics must be determined, two measurements are required on each frequency. In the first, the input impedance of the complete loop probe is measured. In the second, the loop is shorted by clamping a band of copper over the gap. Since the gap width is very small, the impedance of the copper band across the gap is so small compared to the loop impedance that the line is effectively short-circuited. The input impedance, which is now simply that of a shorted transmission line, is measured. The input impedance of a shorted transmission line is given in equation (24). Adding the subscript, f, to the input impedance to designate the frequency, equation (24) is rewritten

$$Z_{(f)} = j\,Z_o \tan\theta. \tag{33}$$

Since the electrical length, θ , is directly proportional to the frequency, the input impedance is

$$Z_{(2f)} = j\,Z_o \tan(2\theta) \tag{34}$$

at twice the frequency designated in equation (33). From standard trigonometric formulas,

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}. \tag{35}$$

Equation (33) is rewritten

$$\tan\theta = \frac{Z_{(f)}}{Z_o} \quad . \tag{36}$$

Equation (34) is rewritten

$$\tan 2\theta = \frac{Z_{(2f)}}{Z_o} \quad . \tag{37}$$

Substituting equations (36) and (37) in equation (35),

$$\frac{Z_{(2f)}}{Z_o} = \frac{2\left(\frac{Z_f}{Z_o}\right)}{1 - \left(\frac{Z_f}{Z_o}\right)^2} \quad . \tag{38}$$

Solving equation (38) for Z_o,

$$Z_o = Z_f \sqrt{\frac{Z_{(2f)}}{Z_{(2f)} - 2\,Z_{(f)}}} \quad . \tag{39}$$

Measured values of the input impedance of the shorted transmission line, at appropriate frequencies, are substituted in equation (39) to determine the characteristic impedance of the line. The value of Z_o obtained by this method is then substituted in equation (33) to determine the electrical length of the line.

Rewriting equation (28) to include the fact that $Z_{01} = Z_{02}$,

$$Z_{in} = Z_{01} \tan\left(\theta_1 + \theta_2 + \arctan\frac{Z_I}{Z_{01}}\right). \tag{40}$$

However, $\theta_1 + \theta_2 = \theta$ of equation (35) and $Z_{01} = Z_o$ of equation (35). These values are substituted in equation (40), and the resulting equation is

$$Z_{in} = Z_o \tan\left(\theta + \arctan \frac{Z_I}{Z_o}\right). \tag{41}$$

Solving equation (41) for Z_I,

$$Z_I = Z_o \tan\left(\arctan \frac{Z_{in}}{Z_o} - \theta\right). \tag{42}$$

Z_I is then calculated for a number of frequencies. If the loop is equivalent to a uniform transmission line, Z_I will conform to equation (24). The procedure shown by equations (33) through (39) is then used to determine the value of the equivalent characteristic impedance and the equivalent electrical length.

To reduce experimental errors and increase accuracy of these calculations, it is desirable that curves of the measured values be plotted and smoothed by graphical or mathematical means before substituting in the equations.

* * *

APPENDIX B

VOLTAGE INDUCED IN A LOOP PROBE BY A SMALL RADIATING LOOP ANTENNA*

It is assumed that a small, circular, single-turn loop-antenna of radius r, has a current, I, flowing in it. The current is assumed to have a uniform amplitude distribution around the circumference of the loop. It is desired to calculate the field strength of the magnetic induction field, designated H, at a point P, which is located at a distance X from the center of the loop. Figure 35 shows the assumed conditions.

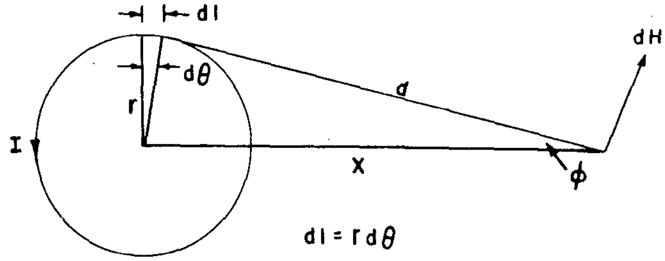


Figure 35

Using rationalized mks units,

$$dH = \frac{I\,dl}{4\pi d^2} = \frac{I\,r\,d\theta}{4\pi d^2}, \tag{43}$$

and

$$dH_x = (\sin\phi)\,(dH) = \frac{r}{d}\,dH = \frac{I r^2 d\theta}{4\pi d^3}. \tag{44}$$

Integrating the equation,

$$H = H_X = \frac{I r^2}{4\pi d^3} \int_0^{2\pi} d\theta, \tag{45}$$

results in

$$H = \frac{I r^2}{2 d^3}. \tag{46}$$

Introducing X,

$$H = \frac{r^2 I}{2(X^2 + r^2)^{3/2}}. \tag{47}$$

---

* Based in part on development by W. O. Swinyard, Proc. I.R.E., 29: 386, July 1941.

Now the electric field strength in volts per meter is related to the magnetic field strength by the equation,

$$E = 120\pi H , \tag{48}$$

whence

$$E = \frac{120\,\pi\, r^2 I}{2\,(X^2 + r^2)^{3/2}} = \frac{188.5\, r^2 I}{(X^2 + r^2)^{3/2}} \tag{49}$$

If it is assumed that $r^2 \ll X^2$,

$$E = \frac{188.5\, r^2 I}{X^3}, \tag{50}$$

where E is in volts per meter, r and X are in meters, and I is in amperes. Converting E to microvolts per meter, r and X to centimeters, and I to milliamperes, the expression becomes

$$E = \frac{18.85\, I r^2 \times 10^6}{X^3} \tag{51}$$

To insure that a pure induction field is developed, it is necessary that

$$X < \frac{\lambda}{2\pi},$$

but it was assumed that

$$r^2 \ll X^2.$$

At a frequency of 400 Mc,

$$\lambda = \frac{300}{400} = 0.75 \text{ meters}$$

$$= 75 \text{ centimeters, and}$$

$$\frac{\lambda}{2\pi} = \frac{75}{2\pi} = 11.94 \text{ centimeters.}$$

It is known that the radius of the radiating loop antenna will be in the order of 2 to 4 centimeters. Therefore, at 400 Mc, r is not much less than X, since X must be less than 11.94 cm. Equation (51) does not apply, and further derivation is necessary.

From equations for mutual inductance, it is known that

$$E_2 = I_1\, w\, M \times 10^3 , \tag{52}$$

where E_2 = the voltage induced in one inductance by the field of a second inductance, microvolts;

I_1 = the current flowing in the second inductance, milliamperes;

M = the mutual inductance between the two inductances, microhenrys; and

$w = 2\pi f$ where f = the frequency of the current, I_1, Mc.

It is also known that the voltage induced in a loop antenna by an r-f field is

$$E_2 = \frac{2\pi E A}{\lambda} = \frac{2\pi E f A}{300}, \tag{53}$$

where E_2 = the voltage induced in the loop, microvolts;

E = r-f field strength, microvolts per meter;

f = frequency of the field, Mc;

λ = wavelength of the field, meters; and

A = area of the loop, square meters.

Equating (52) and (53) and eliminating common terms,

$$2\pi f I_1 M \cdot 10^3 = \frac{2\pi E f A}{300},$$

$$I_1 M \cdot 10^3 = \frac{E A}{300}. \tag{54}$$

But

$$A = \pi r_2^2 \times 10^{-4},$$

where

r_2 = radius of loop probe, centimeters

Therefore,

$$I_1 M \times 10^3 = \frac{E \pi r_2^2 \times 10^{-4}}{300}. \tag{55}$$

Solving for E,

$$E = \frac{3 I_1 M \times 10^5}{\pi r_2^2 \times 10^{-4}} = \frac{3 I_1 M \times 10^9}{\pi r_2^2}. \tag{56}$$

The mutual inductance between two parallel, circular, coaxial, single-turn coils is

$$M = N\sqrt{r_1 \, r_2}\,, \tag{57}$$

where r_1 = radius of radiating loop, centimeters;

r_2 = radius of loop probe, centimeters; and

N = a constant which is a function of the ratio of the shortest and longest distance between points on the circumference of the two loops.

Substituting (57) in (56) yields

$$E = \frac{3 I_1 M x 10^9}{\pi r_2^2} \qquad \frac{3 I_1 N \sqrt{r_1 \, r_2} \; x 10^9}{\pi r_2^2}$$

$$= \frac{3 I_1 \; N x 10^9}{\pi} \sqrt{\frac{r_1}{r_2^3}} = \frac{3 I_1 N x 10^9}{\pi r_2} \sqrt{\frac{r_1}{r_2}} \,. \tag{58}$$

A curve or table is necessary for determining the value of N. Such a curve may be found in Reference 6. However, Table 2 is a tabulation of values of field strength for a current of one milliampere flowing in a radiating loop of 4-cm diameter. Values are tabulated for a number of probes of different diameters and for various spacings between the probe and the radiating loop.

TABLE 2

Field Strength, E, in Millivolts Per Meter, for One Milliampere Current through Loop of 4-cm Diameter*

Spacing Between Loops, X (cm)	Diameter of Loop Probe, cm							
	4	6	8	10	12	14	16	18
6	262.0	226.0	187.0	146.0	117.0	91.9	72.8	58.5
8	120.0	112.0	99.4	85.8	72.6	61.1	55.3	42.9
10	65.9	61.8	56.8	51.0	45.7	40.1	35.1	30.5
12	39.6	37.7	33.8	32.5	29.4	26.8	24.5	22.0
14	25.4	24.2	23.3	21.7	20.4	19.0	17.4	15.9
16	17.0	16.8	15.7	15.6	14.5	13.7	12.5	11.9
18	12.0	11.7	11.8	11.0	10.4	10.1	9.6	9.0
20	8.2	8.7	8.5	8.4	7.8	7.6	7.3	7.1

* To obtain true field strength, multiply E by current through radiating loop in milliamperes.

* * *

APPENDIX C

COMPARISON OF VARIOUS LOOP-ANTENNA EQUATIONS

It is interesting to compare the theoretical developments regarding loop impedance of L. L. Libby, Reference 9, with those of F. M. Colebrook, Reference 10, and R. E. Burgess, References 11 and 12, since part of Appendix A is based on Reference 9.

Colebrook considers an unshielded loop only and makes no attempt to derive actual circuit constants. The equation derived for the effective impedance of the loop is

$$Z_e = 2\,Z \tanh\left(\frac{Pl}{2}\right), \tag{59}$$

where

Z_e = the input impedance of the loop;
Z = the characteristic impedance of the loop;
l = the perimeter of the loop; and

$$P = \sqrt{(R + jwL)jwC}, \tag{60}$$

where

R = the resistance per unit length of the conductor, including any radiation resistance;
L = the inductance per unit length of the conductor;
C = the capacitance per unit length of the conductor; and
$w = 2\pi f$ where f = frequency.

The characteristic impedance is further defined as

$$Z = \sqrt{\frac{R + jwL}{jwC}}, \tag{61}$$

where R, L, C, and w have the same values as defined for equation (60). It should be noted that series resistance is included in the equation, but shunt conductance or parallel resistance is neglected since the dielectric surrounding the loop is assumed to be loss-free.

Colebrook also shows that, when dimensions of the loop are very small compared to wavelength, the equations simplify to the familiar loop equations used at low frequencies.

Burgess, in Reference 12 develops theory for operation of a shielded loop aerial. The development is specifically confined to low frequencies, and all circuit constants are considered lumped with uniform current distribution. This development includes the fallacious assumption that the field can penetrate the shield and induce a voltage on the shielded inner conductors but neglects any capacitance. Burgess states in Reference 11 that the theory of Reference 12 is only a first approximation.

In Reference 11, Burgess develops theory for a shielded loop, both balanced and unbalanced, based on classical transmission-line theory. This development neglects resistance and leakage. The derived equation for the input impedance is similar to that obtained by Libby in Reference 9. The impedance of the "screen" (shield) is obtained from the formula

$$Z = 2Z_{01} \tan B_1 S, \tag{62}$$

where

Z = input impedance,
Z_{01} = characteristic impedance of shield,
$B_1 = \dfrac{2\pi}{\lambda}$,
λ = wavelength in meters, and
S = 1/2 perimeter of loop.

Burgess further derives specific values for Z_{01} based on loop dimensions. The equations derived for a circular loop are given below, without the intermediate steps of the derivation, and are compared with the equation derived by Libby. The characteristic impedanc is obtained from

$$Z_{01} = 30\, L_{11}, \tag{63}$$

where

$$L_{11} = 2\left(\log_e \frac{8R}{ro} - 2\right) \text{ or } 2\left(\log_e \frac{P}{ro} - 1.76\right); \tag{64}$$

R = Radius of shield, centimeters;
ro = Outside radius of a cross-section of the shield; and
P = perimeter of the loop, $2\pi R$.

Combining equations (63) and (64),

$$Z_{01} = 30 \times 2\left(\log_e \frac{8R}{ro} - 2\right). \tag{65}$$

Since the term $2Z_{01}$ appears in equation (62),

$$2Z_{01} = 2 \times 30 \times 2\left(\log_e \frac{8R}{ro} - 2\right)$$

$$= 120\left(\log_e \frac{8R}{ro} - 2\right). \tag{66}$$

Substituting $2.303 \log_{10}$ for $\log_e$,

$$2Z_{01} = 120\left(2.303 \log_{10} \frac{8R}{ro} - 2\right). \tag{67}$$

Libby's equation for loop characteristic impedance, as developed in Reference 9, is

$$Z_0 = 276 \log_{10} \frac{R}{ro}, \tag{68}$$

which may be written

$$Z_0 = 120\left(2.303 \log_{10} \frac{R}{ro}\right). \tag{69}$$

This is similar to equation (67). Changing equation (67) from the form

$$2\,Z_{01} = 120\left(2.303 \log_{10} \frac{8R}{ro} - 2\right)$$

to the form

$$2\,Z_{01} = 120\left(2.303 \log_{10} \frac{R}{ro} - K\right) \qquad (70)$$

will reveal the numerical difference in the characteristic impedance calculated by the two equations.

Rewriting equation (67),

$$2\,Z_{01} = 120\left(2.303 \log_{10} 8 + 2.303 \log_{10} R - 2.303 \log_{10} ro - 2\right), \qquad (71)$$

and rewriting equation (68),

$$2\,Z_{01} = 120\left(2.303 \log_{10} R - 2.303 \log_{10} ro - K\right), \qquad (72)$$

and by equating (71) and (72) and solving for K,

$$120\left(2.303 \log_{10} 8 + 2.303 \log_{10} R - 2.303 \log_{10} ro - 2\right)$$
$$= 120\left(2.303 \log_{10} R - 2.303 \log_{10} ro - K\right)$$
$$K = 2 - 2.303 \log_{10} 8 = 2 - 2.07945 = -\,0.07945.$$

Substituting the value of K in equation (70),

$$2\,Z_{01} = 120\left(2.303 \log_{10} \frac{R}{ro} + .07945\right)$$
$$= 276 \log_{10} \frac{R}{ro} + 9.53. \qquad (73)$$

This equation differs from (68) by the presence of the constant. The value of characteristic impedance calculated from Burgess' equation will be 9.53 ohms greater than that calculated from Libby's equation.

In contrast to the derivation by Colebrook, both Burgess and Libby neglect resistance of the conductor. This appears to be justified, since the resistance of a loop antenna can be shown to be very small compared to its reactance.

The "ohmic" resistance of a round wire or tube is given in Reference 6 as

$$R = \frac{83.2\sqrt{f} \times 10^{-9}}{d}, \qquad (74)$$

where

R = resistance, ohms per centimeter;
f = frequency, cycles; and
d = diameter of wire or tube, centimeters.

Converting f to megacycles in equation (74),

$$R = \frac{83.2\sqrt{f} \times 10^{-6}}{d}. \qquad (75)$$

The loop probe has a shield cross-sectional diameter in the order of one centimeter, so that a resistance of one ohm per centimeter will be obtained at 1.445×10^{8} megacycles. At 400 megacycles the resistance is 2.464×10^{-3} ohms per centimeter.

Radiation resistance is normally calculated from the equation

$$Re = 20B^4A^2, \tag{76}$$

where Re = radiation resistance, ohms;

$B = \frac{2\pi}{\lambda}$;

λ = wavelength, meters; and

A = area of loop, square meters.

This formula is based on assumed uniform current distribution. Horner, in Reference 15, shows that the resistance is slightly higher in actual loops because of nonuniform current distribution but confines his equations to a loop with a perimeter less than a quarter-wavelength, a size of loop still having almost uniform current distribution. The equations of Reference 15 for a circular loop indicate an increase in both radiation and "ohmic" resistance. The radiation resistance of a loop with a perimeter of $\lambda/4$ is increased 65 percent, but the total radiation resistance is only 1.26 ohms. The "ohmic" resistance is increased 41 percent, but again the total is a very small fraction of an ohm. The equations given in Reference 15 are

$$\text{Radiation resistance} = 20B^4\pi 2_r 4 \left[1 + B^2 r^2 \frac{19}{5} + \frac{2\pi^2}{3}\right], \tag{77}$$

$$\text{and "Ohmic" resistance} = 2\pi r R_0 \left[1 + \frac{2}{3}\pi^2 B^2 r^2\right], \tag{78}$$

where $B = \frac{2\pi}{\lambda}$;

λ = wavelength, meters;

r = radius of loop, meters; and

R_0 = resistance per unit length.

The term outside the bracket is the low-frequency value (uniform current distribution) while the term inside the bracket calculates the increase in resistance over that obtained at low frequencies due to nonuniform current distribution

When the perimeter is $\lambda/4$, the reactance of the loop probes considered in this problem will approximate 120 ohms, since the dimensions are such that the characteristic impedance is approximately 275 ohms. The total resistance of such a loop probe, as calculated above, is approximately 1.26 ohms. This resistance is slightly more than 1 percent of the reactance and will have negligible effect on the total input impedance.

It is also of interest that equation (27) of Appendix A, when rewritten to eliminate the "handle" section, becomes the same as Burgess' equation (Reference 11) for the input impedance of an unbalanced shielded loop.

Equation (27) of Appendix A, rewritten, is

$$Z = jZ_{02} \tan\left[\theta_2 + \arctan \frac{(Z_{02} \tan \theta_2 + Z_{0L} \tan \theta L)}{Z_{02}}\right]. \tag{79}$$

Since $\tan (A + B) = \dfrac{\tan A + \tan B}{1 - \text{Tan } A \text{ Tan } B}$, (80)

$$Z = j \frac{Z_{02}\left(\text{Tan } \theta_2 + \dfrac{Z_{02} \tan \theta_2 + Z_{0L} \tan \theta_L}{Z_{02}}\right)}{1 - \left(\dfrac{Z_{02} \text{ Tan}\theta_2 + Z_{0L} \tan \theta L}{Z_{02}}\right)\text{Tan } \theta_2}, \tag{81}$$

$$= jZ_{02} \frac{\dfrac{Z_{02} \tan \theta_2 + Z_{02} \tan \theta_2 + Z_{0L} \tan \theta L}{Z_{02}}}{\dfrac{Z_{02} - Z_{02}(\tan \theta_2)^2 - Z_{0L} \tan \theta L \tan \theta_2}{Z_{02}}}, \tag{82}$$

$$= jZ_{02} \frac{2Z_{02} \tan \theta_2 + Z_{0L} \tan \theta_L}{Z_{02}(1 - \tan^2 \theta_2) - Z_{0L} \tan \theta L \tan \theta_2}, \tag{83}$$

where Z_{0L} = characteristics impedance of loop,
which is = $2Z_{01}$ of Burgess' equation,

Z_{02} = characteristic impedance of coaxial-line section of loop (internal),

and θ_1 & θ_2 = electrical length of line sections.

Burgess' equation for an unbalanced loop is

$$Z_2 = 2j\, Z_{02} \frac{Z_{01} \tan B_1 S + Z_{02} \tan B_2 S}{Z_{02}(1 - \tan^2 B_2 S) - 2\, Z_{01} \tan B_1 S \tan B_2 S}, \tag{84}$$

the same as equation (83) since

$$2\, Z_{01} = Z_{0L}, \quad Z_{02} = Z_{02}, \quad B_1 S = \theta_L, \text{ and } B_2 S = \theta_2.$$

Thus, the developments of several different investigators are found to have substantial agreement.

* * *

UNCLASSIFIED

REFERENCES

1. BuAer ltr. Electronics Division, Aer-EL-41, 36757, dtd. 15 May 1946. Request for assignment of problem to the Naval Research Laboratory, (Problem No. A-282-U).

2. Zepler, E. E. "The technique of radio design." Ch. 8, p. 183-205, London: Chapman and Hall Ltd., 1945.

3. Marchand, N. "Ultrahigh frequency transmission and radiation." Ch. 8, p. 271-279, New York: Wiley, 1947.

4. Bond, D. S. "Radio direction finders." New York: McGraw-Hill, 1944.

5. Sandretto, P. C. "Principles of aeronautical radio engineering." p. 140-142, New York: McGraw-Hill, 1942.

6. Terman, F. E. "Radio engineers handbook." p. 67-68, 813-815, New York: McGraw-Hill, 1943.

7. King, R. W. P., Minno, H. R., and Wing, A. H. "Transmission lines, antennas and wave-guides." p. 231-235, New York: McGraw-Hill, 1945.

8. Hund, A. "Phenomena in high-frequency systems." p. 466-492, New York: McGraw-Hill, 1936.

9. Libby, L. L. "Special aspects of balanced shielded loops." Proc. I. R. E. 34: 641-646, Sept. 1946.

10. Colebrook, F. M. "The application of transmission-line theory to closed aerials." Jour. I. E. E., 83: 403-414, 1938.

11. Burgess, R. E. "Reactance and effective height of screened loop aerials." Wireless Engineer, 21: 210-221, May 1944.

12. Burgess, R. E. "The screened loop aerial." Wireless Engineer, 16: 492-499, Oct. 1939.

13. Sinclair, G., Jordan, E. C., and Vaughn, E. W., "Measurement of aircraft-antenna patterns using models." Proc. I. R. E., 35: 1456, Dec. 1947.

14. Swinyard, W. O. "Measurement of loop antenna receivers." Proc. I. R. E., 29: 386, July 1941.

15. Horner, F. "Properities of loop aerials-non-uniform current distribution." Wireless Engineer, 25: 254-259, August 1948.

PRNC-2400-8-26-49-175